2017

M.Sc.

3rd Semester Examination

APPLIED MATHEMATICS WITH OCEANOLOGY AND COMPUTER PROGRAMMING

PAPER—MTM-304

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Illustrate the answers wherever necessary.

(Numerical Methods and Computer Programming)

Answer *Q. No. 1* and any *four* questions from the rest.

1. Answer any *four* of the following questions : 4×2

(a) Define absolute, relative and percentage errors.

(b) Write a program to solve the following differential equation by Euler's method

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$$

(c) Can Newton-Paphson method be used to solve $f(x) = 0$ if $f(x) = x^{1/3}$? Give reasons.

(d) The expression $x_{n+1} = \frac{3x_n^2 + 2}{8}$ is an iteration scheme to find a root of the equation $f(x) = 0$. Find the function $f(x)$.

(e) Write the error term while finding the interpolating polynomial $p_n(x)$ for $f(x)$ based on $(n+1)$ points $x_0, x_1, \ldots, x_n$.

(f) Derive the trapezoidal formula for integration $\int_a^b f(x)dx$ taking two points in $[a, b]$.

2. Describe finite element method to solve a second order boundary value problem. 8

3. (a) What are the sources of errors ? Describe briefly each.

(b) Calculate $A = \frac{x^3\sqrt{y}}{z^2}$ where $x = 8.36$, $y = 80.46$, $z = 25.8$. The absolute errors in x, y, z are respectively 0.01, 0.02 and 0.03. Find the error in the result. 4+4

4. Find the iterative method based on Newton-Raphson method for finding $\sqrt{N}$ and $N^{1/3}$, where 'N' is a positive real number. Apply the methods for $N = 18$ to obtain the results correct to two significant digits. 8

5. Use Gauss elimination method to find the solution of the following system, so that the iteration converges to the true solution.

$$\begin{aligned} 6x_1 + 2x_2 + 2x_3 &= -2 \\ 2x_1 + 0.6667x_2 + 0.3333x_3 &= 1 \\ x_1 + 2x_2 - x_3 &= 0. \end{aligned}$$ 8

(Perform *four* iterations only)

6. (a) Derive the Newton's divided difference formula while finding the interpolating polynomial $p_n(x)$ for $f(x)$ based on $(n+1)$ points $x_0, x_1, \ldots, x_n$.

(b) Hence apply the above formula to find the interpolating polynomial of $f(x) = 3x^4 + 2x + 5$ at points $x = -1, 0, 1$ and 2. 5+3

7. (a) Derive the Newton's forward difference formula while finding the interpolating polynomial $p_n(x)$ for $f(x)$ based on $(n+1)$ points $x_0, x_1, \ldots, x_n$.

(b) Evaluate $\int_{-2}^{2} |2x+3|\, dx$ using Simson's 1/3 rule with spacing $h = 1$. 4+4

[Internal Assessment —10 Marks]

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