

## **A Study on JELS Inventory Model of Delay-in-Payments Along with Different Mode of Road Transportation under the Carbon Emission Reduction**

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*Received 28 November 2018; accepted 23 January 2019*

### **ABSTRACT**

To create a pollution free environment is a dream for the every mankind. In a modern business world every firms are advised to reduce the pollution from their production and transportation activities. This paper presents an integrated inventory model with the reduction of carbon emission under the different modes of road transportation. And also this proposed model discusses the delay in payments for attracting their customer's. Transportation cost is the function of shipping weight, distance, fuel price and consumption with two types of road transportation modes: i.e., truckload and less than truckload shipment. The Mathematical model develops and optimizes the environmental and economic performances of a supply chain. Finally, a numerical example is given to illustrate the results of the proposed model.

**Keywords:** Logistics; Supply chain coordination; delay in payments; transportation; fuel cost; emission cost

**Mathematical Subject Classification (2010):** 90B05

### **1. Introduction**

In a modern globalization and competitive business environment, companies are interested in improving economic and environmental performances for long term sustainability. Nowadays, the concern of coordination and synchronization between vendor's production cycle and buyer's ordering cycle in supply chain system has larger amount of attention. It has been proved that integrating inventory decisions through determining delivery and production decisions together can reduce total system's cost significantly. In inventory literature, the problem concerning together making lot sizing decisions involving more than one entity in a supply chain is usually known as joint economic lot size (JELS) problems.

This paper analyzes a two echelon inventory system with two modes of truck of varying capacities including fuel cost, the cost of emission from manufacturing and transportation, order cost and setup cost. The two modes of truck transportation are truckload and less than truckload. In truckload transportation, there is a fixed cost per load up to a given capacity. In less than truckload transportation, a delivery with small quantity where cost is calculated by considering base rates, shipping weight, distance and

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discount. A manufacturer can produce single products at a time, but shipments can be made either LTL quantities or TL quantities. In TL transportation mode, a full truckload presents the lot sizes are then dropped off at the respective retail locations from the same transport vehicle, which incurs a fixed shipping charge. In the case of LTL shipping, the shipments are made directly from the supplier to the various retailers individually and the respective shipping cost depend on the amount of load delivered, based on a variable transportation charge. In this inventory model, we consider constant demand rate that must be met without shortages. Each order has a constant holding cost per unit and a setup cost. Riektset al., [18, 19] considered a combination of two different modes of freight transportation: LTL and FTL. The results reveal that the production batch for the TL model is equal to that of the LTL model, but TL is more cost efficient, more quantity loaded than the LTL shipment.

This paper extends the work of Aljazzar et al., [2] and contributed a delay in payments by a buyer after receiving item with strategy to reduce carbon emission along integrated inventory system. In this study, by assuming JELS model by comparing logistic cost with involving two modes of road transportation namely capacitated and incapacitated. The capacitated model is greater than the truckload capacity. Incapacitated model is less than the truckload capacity. The solution procedure is to determine the optimizes to improving the performance of supply chain. Mathematical model and numerical example are presented to illustrate the models. The rest of the paper proceeds as follows, section 2 presents a review of related literature, section 3 defines notations and assumptions, section 4 formulate the mathematical model, section 5 presents numerical example and section 6 conclude the paper.

## 2. Literature review

In 1915, Harris presents the popular economic order quantity. Goyal [8] initially addressed this issue by proposing the Joint Economic Lot Size (JELS) model for coordinating the inventory replenishment decision across the supply chain. One of the main objectives of a supply chain is to improve economic performance [4]. The cost of carbon emissions is generally ignored because sourcing decisions affect the carbon footprint of a supply chain system [5]. Green house gas (GHG) emissions are generated from different sources, of which manufacturing and transport activities are primary sources, are a well-discussed topic in the literature. Many authors have analyzed the volume of transport emissions [3, 6, 7, 10]. Fossil fuel is not only the cause of transport emissions, [17] but also a major cost in transportation.

Delaying payments after receiving goods is a common business practice in supply chains; many authors have worked in this area [20, 23]. A recent study [1] has investigated the impact of permissible delay-in-payments, when treated as a decision variable, on the economic performance of a two-level supply chain for three different production policies/models; namely, [9, 12, 14].

This paper derives two different modes of road transportation 1) Shipments that result in true truckload (TL) shipping quantities. 2) Shipments that are not likely to be over-declared as TL and are therefore shipped at less than truckload (LTL) rates. With TL transportation, a company may over-declare a quantity that uses less than the capacity available and transport this freight at the cost of a full load. One example of TL transportation is a company that uses a truck to transport freight. An example of LTL

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transportation is a company that uses a third party carrier such the United Parcel Service to ship freight. The logistic costs function of [22] were combined to determine an optimal production batch and the number of deliveries for TL and LTL shipments. Rahman et al., [16] to determine an incorporating logistic cost with JELS model. Wangsa [13] described Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions. Mendoza et al., [15] presented an algorithm based on a grossly simplified freight rate structure for truckload or least-than-truckload. The above mentioned paper concerned on developing single manufacturer-retailer model which considers delay in payments, logistic cost, fuel cost, emission cost.

### 3. Notations and assumptions

#### 3.1. Notations

To develop the model we use the following notations:

$D$	: Retailer's demand rate, when $D < P$	Units/year
$P$	: Manufacturer's production rate	Units/year
$n$	: Vendor's cycle	Unit less
$t$	: Interest free permissible delay in payments from the time of receiving a lot	Days
$Q$	: Shipment lot size	Units
$\tau$	: Time for a retailer to settle its account with a supplier. If $\tau > t$ the supplier charges interest for the period of $\tau - t$	Days
$x$	: A subscript corresponding to a supply chain member where $m$ means manufacturer, $r$ means retailer, $s$ means system	unit-less
$A_x$	: Setup cost for player $x$	\$
$C_x$	: Item cost for player $x$	\$/unit
$\psi^x$	: Order cost for the player $x$	\$
$h_x$	: Financial holding cost for player $x$	\$/unit/year
$s_x$	: Physical (storage) holding cost for player $x$	\$/unit/year
$\lambda$	: Shipment lot size multiplier in a vendor cycle	unit less
$k_x$	: Return on investment for player $x$ %	
$T_m$	: Cycle time = $\lambda Q / D$	year
$d_m$	: Distance from manufacturer to the freight	miles
$d_b$	: Distance from freight to the buyer	miles
$w$	: Weight of a unit part	lbs/unit
$\alpha$	: Discount factor for LTL shipments	$0 \leq \alpha \leq 1$
$F_x$	: The freight rate for full truckload (\$/lb/mile)	
$F_y$	: The freight rate for partial load (\$/lb/mile)	
$\delta$	: Fuel price	(\$/liter)
$\gamma$	: Fuel consumed by diesel truck	(liters/mile)
$W_x$	: Full truckload (FTL) shipping weight	(lbs)
$W_y$	: Actual shipping weight	(lbs)
$T_r$	: Retailer's cycle time	year
$T_e$	: Transport emission tax	\$

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$T_{ci}$	:Emission tax rate	\$/Ton
$E_m$	:Manufacturing emissions	g/unit
$T_{cap}$	:Truck capacity	Units/Truck
$\eta$	: Number of trucks per shipment ( $Q/T_{cap}$ )	Integer

### 3.2. Assumptions

- 1) A retailer orders a single product from a manufacturer under the demand is constant over time.
- 2) The manufacturer offers a permissible period (interest-free period) to the retailer to settle its balance.
- 3) Logistic cost with two modes of road transportation – Truckload (TL) and less-than truckload (LTL) shipments with fuel consumption and emissions remains constant over time.
- 4) All the trucks will always use the same path.
- 5) No shortages are allowed. Thus, the production rate is greater than the demand
- 6) Financial cost at the buyer's end is greater than that of the supplier.

### 4. Mathematical model

This paper extends the work of Aljazzar et al., [2] with describes a combined effects of delay in payments and environmental issues on the performance of a supply chain under the capacitated, incapacitated integrated inventory model along with the three cases.

#### 4.1. Capacitated integrated model

**Case 1: where  $0 \leq t = \tau \leq T_r$**

This case represents an interest free permissible delay in payments ( $t$ ) is the same as the time for retailer to settle its account with a supplier ( $\tau$ ). The annual system cost defined as follows,

$$\begin{aligned} \psi_1^s = & \frac{A_r D}{Q} + \frac{A_m D}{\lambda Q} + (C_r + C_m)D + (h_m + s_m) \frac{Q}{2} \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_m D t \\ & + h_r \frac{(Q-Dt)^2}{2Q} + \frac{s_r Q}{2} - C_r D (e^{k_r \tau} - 1) + (C_r - C_m)D (e^{k_m t} - 1) + E_m D T_{ci} \\ & + \frac{D}{Q} (F_x W_x + \delta \gamma) (2d_m + d_b) + \frac{\eta D T_e}{Q} \end{aligned} \quad (1)$$

The optimal value of lot size  $Q$  is

$$Q_1^* = \sqrt{\frac{2D \left( \frac{A_m}{\lambda} + A_r + \frac{h_r D t^2}{2} + (F_x W_x + \delta \gamma) (2d_m + d_b) + \eta T_e \right)}{(h_m + s_m) \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_r + s_r}} \quad (2)$$

**Case 2: where  $0 \leq t < \tau \leq T_r$**

This case represents the retailer settles its balance with supplier beyond the permissible period permitted by the supplier but before receiving the next shipment.

So, the annual system cost becomes,

$$\begin{aligned} \psi_2^s = & \frac{A_r D}{Q} + \frac{A_m D}{\lambda Q} + (C_r + C_m)D + (h_m + s_m) \frac{Q}{2} \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_m D \tau \\ & + h_r \frac{(Q-D\tau)^2}{2Q} + \frac{s_r Q}{2} - C_r D (e^{k_r \tau} - 1) + (C_r - C_m)D (e^{k_m t} - 1) + E_m D T_{ci} \end{aligned}$$

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$$+ \frac{D}{Q}(F_x W_x + \delta\gamma)(2d_m + d_b) + \frac{\eta DT_e}{Q} \quad (3)$$

The optimal value of lot size  $Q$  is

$$Q_2^* = \sqrt{\frac{2D\left(\frac{A_m}{\lambda} + A_r + \frac{h_r D \tau^2}{2} + (F_x W_x + \delta\gamma)(2d_m + d_b) + \eta T_e\right)}{(h_m + s_m)\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + h_r + s_r}} \quad (4)$$

**Case 3: where  $0 \leq t \leq T_r < \tau$**

This case presents when a retailer settles its balance beyond the permitted period and after receiving the next shipment. So the annual system cost becomes,

$$\begin{aligned} \psi_3^s = & \frac{A_r D}{Q} + \frac{A_m D}{\lambda Q} + (C_r + C_m)D + (h_m + s_m) \frac{Q}{2} \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_m D \tau \\ & + \frac{s_r Q}{2} - C_r D (e^{k_r \tau} - 1) + (C_r - C_m)D (e^{k_m t} - 1) + E_m D T_{ci} \\ & + \frac{D}{Q}(F_x W_x + \delta\gamma)(2d_m + d_b) + \frac{\eta DT_e}{Q} \end{aligned} \quad (5)$$

The optimal value of lot size  $Q$  is

$$Q_3^* = \sqrt{\frac{2D\left(\frac{A_m}{\lambda} + A_r + (F_x W_x + \delta\gamma)(2d_m + d_b) + \eta T_e\right)}{(h_m + s_m)\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + s_r}} \quad (6)$$

#### 4.2. Incapacitated integrated model

**Case 1: where  $0 \leq t = \tau \leq T_r$**

This case represents an interest free permissible delay in payments ( $t$ ) is the same as the time for retailer to settle its account with a supplier ( $\tau$ ). So the annual system cost becomes,

$$\begin{aligned} \psi_1^s = & \frac{A_r D}{Q} + \frac{A_m D}{\lambda Q} + (C_r + C_m)D + (h_m + s_m) \frac{Q}{2} \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_m D \tau \\ & + h_r \frac{(Q-D\tau)^2}{2Q} + \frac{s_r Q}{2} - C_r D (e^{k_r \tau} - 1) + (C_r - C_m)D (e^{k_m t} - 1) + E_m D T_{ci} \\ & + \frac{D}{Q}(\alpha F_x W_x + \delta\gamma)(2d_m + d_b) + F_x D (2d_m + d_b) w (1 - \alpha) + \frac{\eta DT_e}{Q} \end{aligned} \quad (7)$$

The optimal value of lot size  $Q$  is

$$Q_1^* = \sqrt{\frac{2D\left(\frac{A_m}{\lambda} + A_r + \frac{h_r D \tau^2}{2} + (\alpha F_x W_x + \delta\gamma)(2d_m + d_b) + \eta T_e\right)}{(h_m + s_m)\left(\frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1\right) + h_r + s_r}} \quad (8)$$

**Case 2: where  $0 \leq t < \tau \leq T_r$**

This case represents the retailer settles its balance with supplier beyond the permissible period permitted by the supplier but before receiving the next shipment. So annual system cost becomes,

$$\begin{aligned} \psi_2^s = & \frac{A_r D}{Q} + \frac{A_m D}{\lambda Q} + (C_r + C_m)D + (h_m + s_m) \frac{Q}{2} \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_m D \tau \\ & + h_r \frac{(Q-D\tau)^2}{2Q} + \frac{s_r Q}{2} - C_r D (e^{k_r \tau} - 1) + (C_r - C_m)D (e^{k_m t} - 1) + E_m D T_{ci} \end{aligned} \quad (9)$$

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$$+ \frac{D}{Q} (\alpha F_x W_x + \delta \gamma) (2d_m + d_b) + F_x D (2d_m + d_b) w (1 - \alpha) + \frac{\eta DT_e}{Q}$$

The optimal value of lot size  $Q$  is

$$Q_2^* = \sqrt{\frac{2D \left( \frac{A_m}{\lambda} + A_r + \frac{h_r D \tau^2}{2} + (\alpha F_x W_x + \delta \gamma) (2d_m + d_b) + \eta T_e \right)}{(h_m + s_m) \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_r + s_r}} \quad (10)$$

**Case 3: where  $0 \leq t \leq T_r < \tau$**

This case presents when a retailer settles its balance beyond the permitted period and after receiving the next shipment. So the annual system cost becomes,

$$\begin{aligned} \Psi_3^s = & \frac{A_r D}{Q} + \frac{A_m D}{\lambda Q} + (C_r + C_m) D + (h_m + s_m) \frac{Q}{2} \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + h_m D \tau \\ & + \frac{s_r Q}{2} - C_r D (e^{k_r \tau} - 1) + (C_r - C_m) D (e^{k_m t} - 1) + E_m D T_{ci} \\ & + \frac{D}{Q} (\alpha F_x W_x + \delta \gamma) (2d_m + d_b) + F_x D (2d_m + d_b) w (1 - \alpha) + \frac{\eta DT_e}{Q} \end{aligned} \quad (11)$$

The optimal value of lot size  $Q$  is

$$Q_3^* = \sqrt{\frac{2D \left( \frac{A_m}{\lambda} + A_r + (\alpha F_x W_x + \delta \gamma) (2d_m + d_b) + \eta T_e \right)}{(h_m + s_m) \left( \frac{2D}{P} + \frac{\lambda(P-D)}{P} - 1 \right) + s_r}} \quad (12)$$

## 5. Numerical example

In this section we provide the numerical example for the proposed model. The values of input parameters are  $D = 1000$  units per year,  $P = 3200$  units per year,  $A_m = 200$  per order,  $C_m = 15$  per unit,  $h_m = 3$  per unit per year,  $s_m = 9$  per unit per year,  $k_m = 0.1\%$ ,  $A_r = 30$  per order,  $C_r = 20$  per unit,  $h_r = 4$  per unit/year,  $s_r = 12$  per unit/year,  $k_r = 0.2\%$ ,  $a = 0.0000003$ ,  $b = 0.0012$ ,  $c = 1.4$ ,  $d = 500$  km,  $T_{cap} = 80$  units,  $T_{ci} = 30$  per ton,  $\alpha = 0.11246$ ,  $F_x = 0.000040217$  \$/lb/mile,  $W_x = 46,000$  lbs,  $\delta = 1.02$  \$/liter,  $\gamma = 0.63569$  liters/mile,  $d_b = 600$ ,  $d_m = 50$ ,  $w = 22$  lbs/unit

### Capacitated integrated model:

Cases	$\lambda$	H	$Q^*$	Manufacturer's Cost	Retailer's Cost	Total cost
Case:1	2	4	377	36,474	27,839	64,313
Case:2	1	5	464	35,463	26,702	62,165
Case:3	1	5	508	35,349	25,168	60,517

### Incapacitated integrated model:

Cases	$\lambda$	H	$Q^*$	Manufacturer's Cost	Retailer's Cost	Total cost
Case:1	2	3	245.8	35,829	25,467	61,296
Case:2	1	4	330.6	35,359	24,438	59,797
Case:3	1	5	464.8	35,910	23,285	59,195

## 6. Conclusion

In this paper we consider a JELS model with delay in payments and by assuming a truck of varying capacities under the strategy to reduce carbon emission. On comparison of TL

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and LTL mode of logistics helps to improve performance of supply chain. From the numerical result, we can able to find that using TL and LTL transportation can reduce the overall total cost of the system.

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