MCA/IS/104/13

MCA 1st Semester Examination, 2013

FOUNDATION IN MATHEMATIC AND LOGIC

PAPER—CS/MCA-104

Full Marks : 70

Time : 3 hours

Answer **Q. No. 1** and **five** from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *five* questions : 2×5

   (*a*) Define Eulerian graph and give an example. $1 + 1$

   (*b*) Explain finite and infinite sets with example. $1 + 1$

   (*c*) Give two examples of non-planar graph. $1 + 1$

(*Turn Over*)

(*d*) Define partition of a set and explain it with example. 1 + 1

(*e*) Define symmetric and skew symmetric matrix. 1 + 1

(*f*) What is limit of a sequence ?Give an example of convergence sequence. 1 + 1

(*g*) What is relation between permutation and combination, gives its expression ? 1 + 1

2. (*a*) Prove that 6

$$\begin{vmatrix} 1+a_1 & 1 & 1 \\ 1 & 1+a_2 & 1 \\ 1 & 1 & 1+a_3 \end{vmatrix} = a_1a_2a_3\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)$$

(*b*) Define tree and forest. Show that any circuit free graph with n vertices and $(n-1)$ edges is a tree. 2 + 4

3. (*a*) What is the relation between co-factor and minor of a determinant. Find inverse of the matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{pmatrix}.$$

Is all square matrix are invertible ? Justify your answer. 1 + 4 + 1

(*b*) Show that the mapping $f : R \to R$ define by $f(x) = x^3, x \in R$ is a bijective. Determine f^{-1}. 4 + 2

4. (*a*) How many different words of five letters can you make from the letters of word 'DEMOCRAT' if every word must contain two different vowels and three different consonants ? 4

(*b*) Prove that a graph is Eulerian graph iff it can be decomposed into disjoint circuits. 4

(*c*) Let *A*, *B*, and *C* are subsets of a universal sets. Then simplify the expression

$$(A \cap B) \cup (A \cap B') \cup (A' \cap B) \cup (A' \cap B')$$ 4

5. (*a*) What are the relation between sequence and series. Find the limit of the sequence $\{x_n\}$. Where $x_n = n^{1/n}$. 2 + 4

(*b*) How many ways could the three boys and four girls be arranged a circular table if the boys must sit together and girls as well ? 4

(*c*) Define regular graph and bipartite graph. 1 + 1

6. (*a*) Prove that by induction principle

$$(x+y)^n = \sum_{k=0}^{n} {}^nC_k \, x^k y^{n-k}.$$

for any positive integer n. 6

(*b*) In a Boolean algebra B. for all $a, b, c \in B$ Show that (*i*) $a + (b + c) = (a + b) + c$

(*ii*) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ 6

7. (*a*) Solve the following recurrence relation using generating function

$$a_n + a_{n-1} - 16_{an-2} + 20_{an-3} = 0$$

for $n \geq 3$ and $a_0 = 0, a_1 = 1, a_2 = -1.$ 6

(*b*) Solve the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 - 3n + 1$ with the initial condition $a_0 = 0, a_1 = 1.$ 6

8. (*a*) State Dijkstra's algorithm for shortest path problem. 5

(*b*) Use Dijkstra's algorithm to find the shortest path between the vertices from *S* to *Z* in the following graph : 7