MCA 1st Semester Examination, 2013 FOUNDATION IN MATHEMATIC AND LOGIC

PAPER -CS/MCA-104

Full Marks: 70

Time: 3 hours

Answer Q. No. 1 and five from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any five questions:

 2×5

- (a) Define Eulerian graph and give an example.
- (b) Explain finite and infinite sets with example. 1 + 1
- (c) Give two examples of non-planar graph. 1+1

(Turn Over)

- (d) Define partition of a set and explain it with example. 1+1
- (e) Define symmetric and skew symmetric matrix. 1+1
- (f) What is limit of a sequence ?Give an example of convergence sequence. 1+1
- (g) What is relation between permutation and combination, gives its expression? 1+1
- 2. (a) Prove that

 $\begin{vmatrix} 1+a_1 & 1 & 1 \\ 1 & 1+a_2 & 1 \\ 1 & 1 & 1+a_3 \end{vmatrix} = a_1 a_2 a_3 \left(1 + \frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3}\right)$

- (b) Define tree and forest. Show that any circuit free graph with n vertices and (n-1) edges is a tree. 2+4
- 3. (a) What is the relation between co-factor and minor of a determinant. Find inverse of the matrix

(Continued)

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$$\begin{pmatrix} 1 & 0 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{pmatrix}.$$

Is all square matrix are invertible? Justify your answer. 1+4+1

- (b) Show that the mapping $f: R \to R$ define by $f(x) = x^3, x \in R$ is a bijective. Determine f^{-1} .
- 4. (a) How many different words of five letters can you make from the letters of word 'DEMOCRAT' if every word must contain two different vowels and three different consonants?
 - (b) Prove that a graph is Eulerian graph iff it can be decomposed into disjoint circuits.
 - (c) Let A, B, and C are subsets of a universal sets. Then simplify the expression

$$(A \cap B) \cup (A \cap B') \cup (A' \cap B) \cup (A' \cap B')$$
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5. (a) What are the relation between sequence and series. Find the limit of the sequence $\{x_n\}$. Where $x_n = n^{1/n}$. 2+4

(Turn Over)

- (b) How many ways could the three boys and four girls be arranged a circular table if the boys must sit together and girls as well?
- (c) Define regular graph and bipartite graph. 1 + 1
- **6.** (a) Prove that by induction principle

$$(x+y)^n = \sum_{k=0}^n {}^nC_k x^k y^{n-k}.$$

for any positive integer n.

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- (b) In a Boolean algebra B. for all $a, b, c \in B$ Show that (i) a + (b + c) = (a + b) + c(ii) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$
- 7. (a) Solve the following recurrence relation using generating function

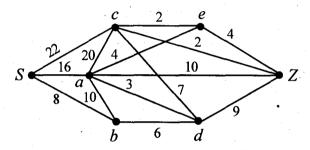
$$a_n + a_{n-1} - 16_{an-2} + 20_{an-3} = 0$$

for $n \ge 3$ and $a_0 = 0$, $a_1 = 1$, $a_2 = -1$.

(b) Solve the recurrence relation $a_n + 5a_{n-1} + 6a_{n-2} = 3n^2 - 3n + 1$ with the initial condition $a_0 = 0, a_1 = 1$.

(Continued)

- **8.** (a) State Dijkstra's algorithm for shortest path problem.
 - (b) Use Dijkstra's algorithm to find the shortest path between the vertices from S to Z in the following graph:



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