PG/IS/ELC-101/12

M.Sc. 1st Semester Examination, 2012

ELECTRONICS

(*Mathematical Methods and Numerical Analysis*)

(Theory)

PAPER–ELC-101

Full Marks : 50

Time : 2 hours

Answer **Q. No. 1** and any **three** questions from the rest

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer *all* questions : 2 × 5

(*a*) Define convolution integral.

(*b*) Find the Fourier transform of $e^{-a|x|}$, when $a > 0$.

(*c*) Explain the analyticity of a complex function in term of Cauchy-Riemann equation.

(*d*) Find the Laplace transform of $t \sin at$ and $e^{kt} \sin at$ from known properties

(*e*) Write the following algebraic expressions in C equivalent form :

(*i*) $(a+b)^{1/2} + \log_e |x+y|$

(*ii*) $y = a^x + e^x (1 + x^2 + \cos x)$.

2. (*a*) Prove the recurrence relation

$$2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$$

where $J_n(x)$ is Bessel function of order n.

(*b*) Show that

$$P_e(-W) = (-1)^l P_e(W).$$

(*c*) Using Laplace transform, solve the following differential equation, –

$$y''(t) - 3y'(t) + 2y(t) = 4e^{2t}$$
with $y(0) = -3, y'(0) = 5$. 3 + 3 + 4

3. (*a*) Prove that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic and find 'v' such that $u + iv$ is analytic.

(*b*) Write down the numerical algorithm of Newton -Raphson method to a transcendental equation. The function $f(x) = e^{2x} - e^x - 2$ has a zero on the interval [0, 1]. Find this zero correct to four significant digits using this method. 4 + (2 + 4)

4. (*a*) Sketch the function $F(t)$ given in terms of unit step function $F(t) = 2U(t) - 3U(t-3) + U(t-4)$ and obtain its Laplace transform.

(*b*) Prove that

$$1 + 3P_1 + 5P_2 + \cdots + (2n+1)P_n = \frac{d}{dz}(P_{n+1} + P_n)$$

where P_n is the Legendre Polynomial.

(*c*) Write the following algebric expressions in C equivalent form

(*i*) $(a+b)^{1/2} + \log_e |x+y|$

(*ii*) $y = a^x + e^x (1 + x^2 + \cos x)$. (2 + 2) + 4 + 2

5. (*a*) Find a root of the equation $x^2 + x - 7 = 0$ by bisection method correct upto two decimal places. Assume that one root lies between 2·188 and 2·204.

(*b*) Write a program in C to compute the value of the series

$$J_0(x) = 1 - \frac{x^2}{2^2.1!1!} + \frac{x^4}{2^4.2!2!} - \frac{x^6}{2^6.3!3!} + \cdots$$

for a given value of x by direct summation of successive terms upto and including the first term that has a magnitude greater than 10^{-8}. 5 + 5

6. (*a*) Evaluate $y(1{\cdot}1)$ using Runge-Kutta method of order 4 for initial value problem

$$\frac{dy}{dx} = x^2 + y$$

and given $y(1) = 0$ and $x = 0{\cdot}1$.

(*b*) Evaluate

$$\int_0^6 \frac{dx}{1+x^2}$$

by using Simpson's $\frac{1}{3}$rd rule, correct to four decimal places dividing the interval (0, 6) into six parts each of width $h = 1$. 5 + 5

[*Internal Assessment* : 10 Marks]