Chapter 1

General introduction and organization of the thesis

1.1 General introduction

'Soft set theory' is a modern mathematical approach for handling *uncertainty* by using the idea of parameterization. Now, the question is, what is uncertainty? Uncertainty is a situation where, the given data or given information are not precisely defined or where, there is a vague or nonspecific information in the given data set. For instance, suppose, a patient has different symptoms with different degrees of belongingness. In that case, it is difficult to a doctor to give a satisfactory illustration about the current status of the patient. This is an example of uncertainty in the field of medical science. Moreover, description of tomorrow's weather forecast is an another example of uncertainty in the field of meteorology. Indeed, in our every-day-life problems in various fields (like, medical science, engineering, environmental science, social science, economics, etc.), uncertainty is a common word to be handled sincerely. Therefore, to give an appropriate justification of real-life based problems, a lot of research is needed under uncertainty.

In general, based on the origin of uncertainty, it can be categorized into two types [75, 85], ***(i) Aleatory uncertainty (ii) Epistemic uncertainty***, as given in Figure 1.1 Aleatory uncertainty occurs from natural inherent randomness in a system. This type of uncertainty can not be avoided or reduced from a system during process. For example, suppose a coin is tossed finite number of times. Then, head or tail will come up with some randomness. But, we can not predict here the exact result for a particular turn, whether, we do lot of experiments. This is an example of aleatory uncertainty. On the other hand, epistemic uncertainty occurs from the lack of knowledge which can be eliminated or reduced from a system. For example, machining problem in a system, error in an experiment for insufficient data, etc. are some examples of epistemic uncertainty. Aleatory uncertainty is usually handled by probability theory and epistemic uncertainty is basically dealt with fuzzy set theory.

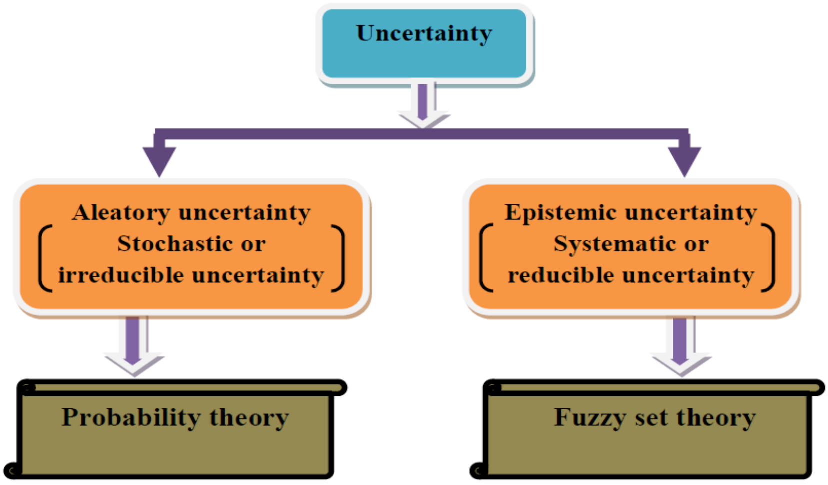

Figure 1.1: Various uncertainties and their solving tools

Probability theory is a traditional mathematical idea which is based on the Aristotelian two-valued logic i.e., yes or no type. In probability theory, we have derived the probability of an event in terms of some numerical value in the interval $[0, 1]$ which is in crisp sense. But, in Zadeh's [183] fuzzy set theory, a vague or imprecise information is defined very precisely by using membership function in terms of some numerical number, lies in the interval $[0, 1]$. Therefore, fuzzy set theory is not only a challenging issue to probability theory, it is actually a generalization of Aristotelian two-valued logic as illustrated in Figure 1.2. In fuzzy set theory, the concept of membership grade of an element is a matter of degree not a matter of affirmation. This is the main interpretation of fuzzy set theory. In the pioneer article [183], Prof. Zadeh said that, *'classical set theory can not handle human behavior properly, where as fuzzy set theory can deal human conceptual process very precisely'*. Consequently, research on fuzzy set theory has been enhanced in a productive way. In many different brunches of science such as, in data analysis, control theory, computational intelligence, artificial intelligence etc., fuzzy set theory have been used very successfully.

At the initial stage, researchers developed some basic operations including, complement, intersection, union, aggregation, etc. through Zadeh's [183] fuzzy set theory. Further, some basic theoretical areas including group theory, ring, field, graph theory, topology, etc. have been defined in terms of fuzzy set theory. In general, in fuzzy set, the description of an element is presented via membership function where, the membership grade of the element indicates the degree of satisfaction. However, in reality, $non\ membership\ degree$ of an element may not always equals to $1 - membership\ degree$

because, hesitation degree may exist in the system. For example, suppose, membership degree ($T(x)$) of a fuzzy set indicates the number of peoples who have given vote to the elected government. Then in that case, there are some peoples who have not given vote to the elected government (this is the non membership degree $F(x)$) and there are also some peoples who have not given their vote at all in that election (this is the hesitation degree). Then, to solve these problems, Atanassov [18, 19] introduced the definition of intuitionistic fuzzy set theory where, $membership\ (T(x)) + non\ membership\ (F(x)) \leq 1$ where, $T(x), F(x) \in [0, 1]$ and $degree\ of\ hesitaion\ = 1 - T(x) - F(x)$. Further, Smarandache generalized this intuitionistic fuzzy set theory to neutrosophic set theory [149, 150] by taking degree of membership ($T(x)$), degree of hesitation ($I(x)$) and degree of non membership ($F(x)$) independently and $T(x), I(x), F(x) \in [0, 1]$ and $0 \leq T(x) + I(x) + F(x) \leq 3$.

However, membership grade of an object or an element in a problem in is itself vague or fuzzy. Then, to handle with these problems, the notion of type-2 fuzzy set [184] has been introduced where, every element of an universal set has two different membership grades; one is primary membership grade and another one is secondary membership grade. Basically, type-2 fuzzy set is employed for solving uncertainty in a type-1 fuzzy set. Therefore, type-2 fuzzy set might be more adequate than the type-1 fuzzy set or general fuzzy set. But in reality, construction of type-2 fuzzy set is very difficult. Further, Mendel [115] proposed the idea of interval type-2 fuzzy set (IT2FS) where, every secondary membership is equal to 1 and then, the membership of an element in a type-2 fuzzy set takes the form as an interval.

In the previous discussions, all the corresponding membership magnitudes (membership or truth membership, non membership or false membership, indeterminate membership) are real-valued, whose values are restricted in the interval $[0, 1]$. Then, in 2001, Romot et al. [137] proposed a new extension of Zadeh's fuzzy set theory, named as complex fuzzy set theory, by using complex fuzzy membership function. Basically, it is an extension of real valued fuzzy membership function to complex fuzzy membership function so that, the range of the fuzzy membership function $[0, 1]$ is transformed into the unit circle of a complex plane. A complex fuzzy membership grade is in terms of complex fuzzy number which takes the form as, $a(x)e^{ib(x)}$, where, $a(x) \in [0, 1]$ is called the amplitude part (fuzzy-valued) and $b(x)$ is called the phase part (real-valued). In many day-to-day life problems, two information need to be expressed together. For example, in periodic or recurring phenomena related problems (which are generally seems in solar activity system, financial indicator system, signal processing system, disease diagnosis system, etc.), time function is a vital part and should be considered additionally with any associated criteria. Such type of problems can be limned through this complex fuzzy set. After Romot's work, Zhang et al. [185] offered some basic operations and properties on complex fuzzy sets including union, intersection, complement, linear sum, etc., when the range of the phase term is restricted in the closed interval $[0, 2\pi]$. After that, Alkouri and Salleh [8] proposed complex intuitionistic fuzzy set by considering both of the membership and non membership degree in complex fuzzy sense. Furthermore, Ali and Smarandache [12] considered complex-valued truth membership, complex-valued indeterminate membership and complex-valued false membership independently to produce the notion of complex neutrosophic set.

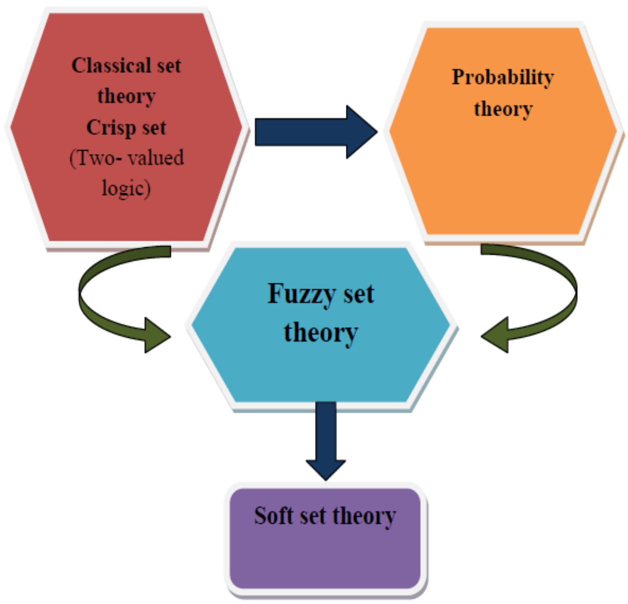

Figure 1.2: Evolution of soft set theory

However, decision makers may not be efficient in providing quantitative information rather they are more effective in offering qualitative information. For instance, to define the 'quality of a dress', the terms 'bad', 'fair', 'good', 'not fair', etc. are more useful than any numerical value. Mathematically, these terms are called linguistic variables. In 1975, Porf. Zadeh [184] developed the concept of a linguistic approach and used it in solving problems in different fields. After that, several researchers have worked on linguistic valued approach [31, 55, 57, 77, 184]. Basically, there are four processes to receive solution from a linguistic based problem such as, (i) extension principle based process (ii) symbolic based computational process (iii) 2-tuple linguistic representation based process and (iv) linguistic scale function based process.

Though fuzzy set theory and its several extensions (intuitionistic fuzzy set, neutrosophic set, complex fuzzy set, linguistic valued set, etc.) are very influential for handling uncertainty, but sometimes, they could not be practically utilized. Prof. D.Molodtsov [118] brought out some difficulties from these existing approaches as follows:

- Probability theory can only handle with stochastically stable phenomena where, for every test, a large number of trials have to be performed. Therefore, it is appropriate for small sample based problems but not for all types of problems.
- In fuzzy set theory, an element is defined by a membership function. But, there does not exist any unique approach for defining a membership function so, it may be varied from one to another. Therefore, there is a lack of uniqueness in defining a proper membership function. Moreover, in a particular case, construction of a membership

function is difficult.

He said that, inadequacy of the parameterization is the main flaw which reduces the applicability of the existing theories (fuzzy set, complex fuzzy set, interval type-2 fuzzy set, etc.). Then, he appointed uncertainty from a different points of view by using the idea of parameterization and proposed the concept of **'soft set theory'** [118]. In soft set theory, an element is defined by using some parameters with out introducing a membership function i.e., in that case, construction of membership function has been replaced by the idea of parameterization for defining an element. Moreover, in soft set theory, no need to to perform a large number of trials to define an alternative. Here, at the initial stage, alternatives are defined approximately based on the human cognitive process. Besides, in soft set theory, parameter consideration process is too euphemistic so that, one can select parameters not only with the help of words, sentence, mapping, etc. but also with the help of uncertain environments like, fuzzy, complex fuzzy, etc. These facilities make this theory more flexible, convenient and more easy to practice. Consequently, with a very small duration of time, soft set theory has become a herculean theoretical approach for solving uncertainty.

Soft set theory has achieved a high peak popularity since its inauguration period. Researchers have developed several classical algebraic properties [11, 107, 146] including, union, intersection, complement, difference, etc. though soft set theory so that it can be utilized in different real-life fields. Moreover, some important algebratic structures such as, relation [111], function [21], group [16], BCK/BCI-algebra [83], semiring [62], etc. have also been initiated on soft set theory. Furthermore, by using parameters under different uncertain environments including fuzzy, intuitionistic fuzzy, neutrosophic, linguistic, complex fuzzy etc., researchers have made several new generalizations of soft set theory such as, fuzzy soft set, [104], intuitionistic fuzzy soft set, linguistic valued soft set [153], [105], complex fuzzy soft set [158], etc. Actually, in these extensions of soft set theory, the issues fuzzyness, intuitionistic fuzzyess, neutrosophicness, etc. can also be dealt through human cognitive process without considering any membership function. Therefore, it is very useful in practice. In existing studies, we have observed that, researchers have been benefitted by these soft set theories in solving real-life problems in different fields like, pattern recognition [65], data analysis [132], medical science [22, 50], specially in decision-making [61, 106].

Soft set theory is a newly proposed mathematical idea for handling uncertainty. Based on some existing articles, we have seen that, soft set theory has a huge application in different fields such as in medical diagnosis, supplier selection, decision making etc. under different uncertain environments such as, fuzzy, intuitionsitic fuzzy, neutrosophic, complex fuzzy, complex neutrosophic etc. So, there is an enormous scope on work on soft set theory. Moreover, similarity measure, distance measure, entropy measure, knowledge measure, etc. are some beneficial ideas by which researchers have solved various types of decision-making problems. Besides, some of these ideas have been defined on soft set theory and used in decision-making. So, if we will develop these ideas for other generalizations of soft set theory, then we can solve more complicated real-life problems. Therefore, in this thesis we have worked on soft set theory and its several extensions and also solved some real-life decision-making problems by introducing various similarity measures and distance measures.

1.2 Basic concepts and terminologies

1.2.1 Fuzzy set (FS) [183].

Let, $A = (X, \mu_A)$ be a fuzzy set over a universal set X where, μ_A is a mapping, called the membership function, defined as, $\mu_A : X \to [0, 1]$. Then, $\forall x \in X$, $\mu_A(x) \in [0, 1]$ is said to be the membership degree or membership grade of an element x in X.
If, $X = \{x_1, x_2, .., x_m\}$ be a universal set, then a fuzzy set A over X is defined as follows:

$$A = \{(x_1, \mu_A(x_1)), (x_2, \mu_A(x_2)), .., (x_m, \mu_A(x_m))\}.$$

1.2.2 Intuitionistic fuzzy set (IFS) [18].

In an intuitionistic fuzzy set, each of the elements has both the membership degree and non membership degree where, the sum of the membership degree and non membership degree is either equals 1 or less than 1. Mathematically, an intuitionistic fuzzy set $\tilde{A}$ is defined as following,

$$\tilde{A} = \{(x, \mu(x), \nu(x))|\forall x \in X\}$$

where $\mu(x) \in [0, 1]$ is the membership degree of $x \in X$ and $\nu(x) \in [0, 1]$ is the non membership degree of $x \in X$ with $0 \leq \mu(x) + \nu(x) \leq 1$.

1.2.3 Neutrosophic set (NS) [150].

A neutrosophic set $\tilde{\tilde{A}}$ over X characterizes every element of X through three individual independent membership magnitudes such as, truth membership, false membership and indeterminate membership. Mathematically, it is defined as follows:

$$\tilde{\tilde{A}} = \{(x, (T(x), I(x), F(x)))|\forall x \in X\}$$

where $T(x), I(x), F(x) \in [0, 1]$ are the truth membership, indeterminate membership and false membership of x and $0 \leq T(x) + I(x) + F(x) \leq 3; \forall x \in X$.

1.2.4 Type-2 fuzzy set (T2FS) [116].

In a type-2 fuzzy set $\bar{A}$, elements are defined by using type-2 membership function as follows,

$$\bar{A} = \{((x, V), \mu(x, V))| \ \forall x \in X, V \in J_x \subseteq [0, 1]\},$$

where, x is the primary variable, J_x is the primary membership of x and $V \in J_x \subseteq [0,1]$ is the secondary variable and $\mu(x, V) \in [0, 1]$ indicates the secondary membership of an element x.

1.2.5 Complex fuzzy set (CFS) [137].

A complex fuzzy set $\tilde{C}$ over a universal set X is defined by taking complex fuzzy membership $(\mu_{\tilde{C}}(x))$ to each of the elements of the set. Its mathematical form is as follows: $\forall x \in X$,

$$\mu_{\tilde{C}}(x) = a_{\tilde{C}}(x)e^{ib_{\tilde{C}}(x)}, i = \sqrt{-1}$$

where, $a_{\tilde{C}}(x) \in [0, 1]$ (fuzzy-valued) is referred to as the amplitude part and $b_{\tilde{C}}(x)$ (real-valued) is referred as the phase part in the complex fuzzy membership $\mu_{\tilde{C}}(x)$ of x. Graphical representation of a complex fuzzy set has been given in Figure 1.3.

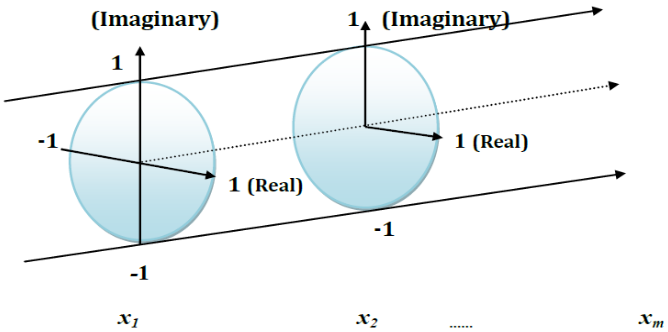

Figure 1.3: Graphical representation of a CFS

In 2009, Zhang et al. [185] studied some basic operational laws on complex fuzzy sets when, range of the phase part is limited in the closed interval $[0, 2\pi]$. In this thesis, we have followed this same range for the phase part to reduce the complexity in computational process.

1.2.6 Complex neutrosophic set (CNS) [12].

In a complex neutrosophic set, every element $x \in X$ has three independent membership magnitudes (such as, truth membership, indeterminate membership and false membership) in complex sense. Mathematically, a complex neutrosophic set $\tilde{C}_N$ over X is defined as follows:

$$\tilde{C}_N = \{(x, (T_N(x), I_N(x), F_N(x)))|x \in X\}$$

where $T_N(x)$, $F_N(x)$ and $I_N(x)$ are the complex-valued truth membership, complex-valued false membership and complex-valued indeterminate membership of $x \in X$ where,

$$T_N(x) = p_N(x)e^{iu_N(x)}, I_N(x) = q_N(x)e^{iv_N(x)}, F_N(x) = r_N(x)e^{iw_N(x)}.$$

Here, $p_N(x), q_N(x), r_N(x)$ are the amplitude parts and $u_N(x), v_N(x), w_N(x)$ are the phase parts of $T_N(x)$, $I_N(x)$ and $F_N(x)$ respectively. Each of the entries is real-valued and $p_N(x)$, $q_N(x)$, $r_N(x) \in [0, 1]$, $u_N(x)$, $v_N(x)$, $w_N(x) \in [0, 2\pi]$ and $0 \leq p_N(x) + q_N(x) + r_N(x) \leq 3$.

1.2.7 Linguistic term set [171].

A, linguistic term set contains some linguistic variables. It can be written as in discrete form like, $S = \{s_\beta | \beta = -\xi, .., -1, 0, 1, .., \xi\}$ where, s_0 is the mid-level term indicating '*indifferent linguistic evaluation*' and the first term ($s_{-\xi}$) and last term (s_ξ) of S represents the '*lower bound*' and '*upper bound*' of the set S.
Further, to deal with all types of linguistic variables, one can use the extended the linguistic term set of S which is named as, continuous linguistic term set $\bar{S}$ where, $\bar{S} = \{s_x | -\xi \leq x \leq \xi\}$.
If $s_x \in S$, then this term is said to be the original linguistic term and otherwise it is referred to as the virtual linguistic term. The sets S and $\bar{S}$ satisfies following axioms:
(1) ***Negative operation property:*** $neg(s_x) = s_{-x}$, $neg(s_0) = s_0$.
(2) ***Order relation property:*** $s_x \leq s_y$ if $x \leq y$.

1.2.8 Soft set (SS) [118].

The concept of soft set theory is a modern mathematical approach for handling uncertainty by using the idea of parameterization where, at the initial stage, elements are defined approximately with respect to some parameters.
Mathematically, a soft set can be defined as follows:
If, X be an initial universe and E be a set of corresponding parameters then, a soft set over the universe X is represented as an order pair (f, E) where, f is a mapping such that, $f : E \to P(X)$.

Example 1.1. Suppose, a universal set contains three dresses as, $X = \{d_1, d_2, d_3\}$, which **Mr. Ruhan** wants to buy. Now, consider that, his corresponding choice parameters are, $E = \{good\ design(e_1), comfortable(e_2), cheap(e_3)\}$.
Now assume that, f is a mapping such that,
$f(e_1) = \{d_1, d_3\}$; $f(e_2) = \{d_1, d_2, d_3\}$; $f(e_3) = \{d_2, d_3\}$.
Then, $(f, E) = \{(e_1, (d_1, d_3)), (e_2, (d_1, d_2, d_3)), (e_3, (d_2, d_3))\}$ is a soft set over X represents the 'attractiveness of the three dresses over the three considered parameters'. Tabular form of (f, E) has been provided in Table 1.1. In this table, the entry 1 is used to represent the belongingness of a parameter to a dress and the entry 0 is used to represent the not belongingness of a parameter to a dress. In Figure 1.4 we have explained this example graphically.

1.2.9 Fuzzy soft set (FSS) [22].

A FSS over a universal set X is depicted as an order pair $((\tilde{f}, E))$ of a mapping $\tilde{f}$ and a parameter set E over a universal set where, $\tilde{f}$ is defined as, $\tilde{f} : E \to \tilde{P}(X)$.
Then, $\forall e_j \in E$, $(\tilde{f}, E) = (e_j, \tilde{f}(e_j)) = \{(e_j, (x_s, \tilde{f}_{e_j}(x_s))) | \forall x_s \in X\}$,

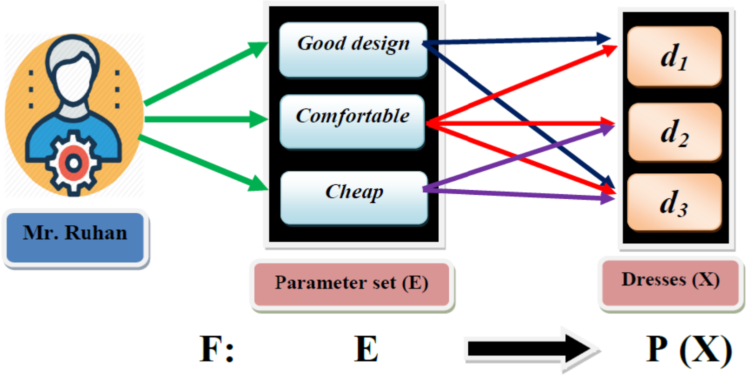

Figure 1.4: Description of a soft set through Example 1.1

where, $\tilde{f}_{e_j}(x_s)$ is a fuzzy rating of an alternative $x_s \in X$ over a parameter $e_j \in E$.

Example 1.2. Assume that, $X = \{D_1, D_2, D_3\}$ be the universal set contains three dresses and $E = \{good\ design(e_1),\ comfortable(e_2),\ cheap(e_3)\}$ be the parameter set contains three parameters which are in fuzzy sense. Now, assume a mapping $\tilde{f}$ to illustrates '*the attractiveness of the three dresses over the three parameters*' as given following: $\tilde{f}(e_1) = \{(D_1, 0.1), (D_2, 0.4), (D_3, 0.5)\}$; $\tilde{f}(e_2) = \{(D_1, 0.3), (D_2, 0.1), (D_3, 0.4)\}$; $\tilde{f}(e_3) = \{(D_1, 0.4), (D_2, 0.6), (D_3, 0.3)\}$.
Then,
$(\tilde{f}, E) = \{(e_1, \{(D_1, 0.1), (D_2, 0.4), (D_3, 0.5)\}), (e_2, \{(D_1, 0.3), (D_2, 0.1), (D_3, 0.4)\}),$ $(e_3, \{(D_1, 0.4), (D_2, 0.6), (D_3, 0.3)\})\}$ is a fuzzy soft set over X which represents '*the attractiveness of the three dresses over the three parameters*'. In Table 1.2, its tabular form has been given.

Table 1.1: SS (f, E) based on Example 1.1

	e_1	e_2	e_3
d_1	1	1	0
d_2	0	1	1
d_3	1	1	1

Table 1.2: FSS $(\tilde{f}, E)$ based on Example 1.2

	e_1	e_2	e_3
D_1	0.1	0.3	0.4
D_2	0.4	0.1	0.6
D_3	0.5	0.4	0.3

1.2.10 Intuitionistic fuzzy soft set (IFSS) [105].

An IFSS is an extension of fuzzy soft set where, each of elements of X is illustrated via membership degree and non membership degree with respect to some considered

parameters. Mathematically, an IFSS is defined as follows:
$(\tilde{\tilde{f}}, E) = (e, \tilde{\tilde{f}}(e)) = \{(e_j, (x_s, \tilde{\tilde{f}}_{e_j}(x_s)))|\forall x_s \in X\}$
where, $\tilde{\tilde{f}}_{e_j}(x_s)$ is the intuitionistic fuzzy rating of an alternative $x_s \in X$ over a parameter $e_j \in E$.

Example 1.3. Now consider a universal set as, $X = \{y_1, y_2, y_3\}$ and some corresponding parameters as, $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ which are in intuitionistic fuzzy sense. Now, assume a mapping $\tilde{\tilde{f}}$ over the set X as follows:
$\tilde{\tilde{f}}(\varepsilon_1) = \{(y_1, (0.7, 0.1)), (y_2, (0.5, 0.4)), (y_3, (0.2, 0.4))\}$;
$\tilde{\tilde{f}}(\varepsilon_2) = \{(y_1, (0.3, 0.4)), (y_2, (0.7, 0.2)), (y_3, (0.8, 0.1))\}$;
$\tilde{\tilde{f}}(\varepsilon_3) = \{(y_1, (0.1, 0.6)), (y_2, (0.6, 0.3)), (y_3, (0.7, 0.3))\}$.
Then, $(\tilde{\tilde{f}}, E) = \{(\varepsilon_1, ((y_1, (0.7, 0.1)), (y_2, (0.5, 0.4)), (y_3, (0.2, 0.4)))), (\varepsilon_2, ((y_1, (0.3, 0.4)), (y_2, (0.7, 0.2)), (y_3, (0.8, 0.1)))), (\varepsilon_3, ((y_1, (0.1, 0.6)), (y_2, (0.6, 0.3)), (y_3, (0.7, 0.3))))\}$ is an intuitionistic fuzzy soft set over X. In Table 1.3, it has been represented in tabular form.

Table 1.3: IFSS $(\tilde{\tilde{f}}, E)$ based on Example 1.3

	ε_1	ε_2	ε_3
y_1	$(0.7, 0.1)$	$(0.3, 0.4)$	$(0.1, 0.6)$
y_2	$(0.5, 0.4)$	$(0.7, 0.2)$	$(0.6, 0.3)$
y_3	$(0.2, 0.4)$	$(0.8, 0.1)$	$(0.7, 0.3)$

1.2.11 Complex fuzzy soft set (CFSS) [158].

A CFSS over a universal set X is defined as an order pair $(\tilde{\tilde{F}}, E)$ where, $\tilde{\tilde{F}}$ is a function defined as, $\tilde{\tilde{F}} : E \to \tilde{\rho}(X)$. Here, $\tilde{\rho}(X)$ is the set of all complex fuzzy subsets of the set X.

Let, $X = \{y_1, y_2, .., y_m\}$ be the set of objects as a universal set and $E = \{\varepsilon_1, \varepsilon_2, .., \varepsilon_n\}$ be the set of some parameters which are in complex fuzzy sense, then, a complex fuzzy soft set $(\tilde{\tilde{F}}, E)$ over the universal set X is defined as follows:

$$\begin{aligned}(\tilde{\tilde{F}}, E) &= \{(\varepsilon_1, \tilde{\tilde{F}}(\varepsilon_1)), (\varepsilon_2, \tilde{\tilde{F}}(\varepsilon_2)), .., (\varepsilon_n, \tilde{\tilde{F}}(\varepsilon_n))\} \\ &= \{(\varepsilon_1, ((y_1, p_{11}e^{iu_{11}}), (y_2, p_{21}e^{iu_{21}}), .., (y_m, p_{m1}e^{iu_{m1}}))), (\varepsilon_2, ((y_1, p_{12}e^{iu_{12}}), \\ &(y_2, p_{22}e^{iu_{22}}), .., (y_m, p_{m2}e^{iu_{m2}}))), .., (\varepsilon_n, ((y_1, p_{1n}e^{iu_{1n}}), (y_2, p_{2n}e^{iu_{2n}}), .., \\ &(y_m, p_{mn}e^{iu_{mn}})))\}\end{aligned}$$

where, $p_{sj} \in [0, 1]$ is the amplitude part and $u_{sj} \in [0, 2\pi]$ is the phase part of the evaluation of an alternative y_s, $s = 1, 2, .., m$ with respect to a parameter ε_j, $j = 1, 2, .., n$.

Example 1.4. In medical science, a common symptom may be the caused of various diseases. Moreover, various diseases may have a common symptom. Now, to express the availability of some symptoms over some diseases, we have used complex fuzzy soft set where, diseases have been assumed as the elements of a universal set and associated symptoms have been considered as the set of parameters. So, the satisfaction degree of a symptom to a disease is in terms of complex fuzzy membership where, '*affected level or belongingness level of a symptom*' to a disease is described through the amplitude part and '*duration of time of a symptom*' for the disease is described through the phase part.

Now, consider three diseases as, X={Viral fever (x_1), Food poisoning (x_2), Diphtheria (x_3)} and some related symptoms of these three diseases are, E={fever (s_1), breathing difficulty (s_2), body pain (s_3), weight loss (s_4), cough (s_5)}.

Now let, $(\tilde{\tilde{F}}, E)$ be a complex fuzzy soft set over X which represents the availability of the five symptoms over the three diseases as given in Table 1.4. All the data have been taken based on 15 days. Then, to elicit all the data through complex fuzzy membership, 2π has been taken instead of 15 days.

Table 1.4: CFSS $(\tilde{\tilde{F}}, E)$ based on Example 1.4

	s_1	s_2	s_3	s_4	s_5
x_1	$1e^{i2\pi}$	$0.2e^{i\pi/4}$	$0.2e^{i6\pi/7}$	$0.7e^{i\pi/4}$	$0.1e^{i\pi}$
x_2	$0.6e^{i2\pi/3}$	$0.1e^{i\pi/4}$	$0.5e^{i\pi}$	0	$0.6e^{i\pi/6}$
x_3	$0.8e^{i\pi}$	$0.3e^{i\pi/6}$	$0.1e^{i3\pi/7}$	$1e^{i2\pi}$	$0.5e^{i\pi/2}$

1.2.12 Complex neutrosophic soft set (CNSS) [29].

A CNSS over X is defined as an order pair $(f_{\tilde{C}_N}, E)$ where, $f_{\tilde{C}_N}$ a mapping such that, $f_{\tilde{C}_N} : E \to \tilde{\tilde{\rho}}(X)$ and $\tilde{\tilde{\rho}}(X)$ is the set of all complex neutrosophic subsets of the set X. Mathematical it is defined as follows: $\forall e \in E$,

$$(f_{\tilde{C}_N}, E) = (e, f_{\tilde{C}_N}(e)) = \{(x, (T(e)(x), I(e)(x), F(e)(x))) \mid \forall\ x \in X\},$$

where, $T(e)(x), I(e)(x), F(e)(x)$ are the complex-valued truth membership, complex-valued indeterminate membership and complex-valued false membership of an element $x \in X$ with respect to a parameter e where, $T(e)(x) = p(e)(x)e^{iu(e)(x)}$, $I(e)(x) = q(e)(x)e^{iv(e)(x)}$ and $F(e)(x) = r(e)(x)e^{iw(e)(x)}$.
Here, all the amplitude parts $p(e)(x), q(e)(x), r(e)(x)$ are restricted in the interval $[0, 1]$ with $0 \leq p(e)(x) + q(e)(x) + r(e)(x) \leq 3$ and all the phase parts $u(e)(x), v(e)(x), w(e)(x)$ are restricted in the interval $[0, 2\pi]$.

Example 1.5. Let us consider a universal set as, $X = \{x_1, x_2, x_3\}$ and some complex neutrosophic sense based parameters as, $E = \{e_1, e_2, e_3\}$. Now, assume a complex neutrosophic soft set $(f_{\tilde{C}_N}, E)$ over X as given in Table 1.5.

Table 1.5: CNSS $(f_{\tilde{C}_N}, E)$ based on Example 1.5

	e_1	e_2	e_3
x_1	$(1e^{i2\pi}, 0e^{i0}, 0e^{i0})$	$(0.2e^{i\pi/8}, 0.4e^{i\pi/5}, 0.4e^{i2\pi/5})$	$(0.2e^{i6\pi/7}, 0.5e^{i\pi/5}, 0.2e^{i7\pi/9})$
x_2	$(0.6e^{i2\pi/3}, 0.2e^{i\pi/3}, 0.1e^{i\pi/3})$	$(0.1e^{i\pi/4}, 0.6e^{i\pi/10}, 0.3e^{i3\pi/7})$	$(0.5e^{i\pi}, 0.1e^{i\pi/5}, 0.4e^{i\pi/3})$
x_3	$(0.8e^{i\pi}, 0e^{i0}, 0.2e^{i\pi/7})$	$(0.3e^{i\pi/6}, 0.4e^{i4\pi/5}, 0.3e^{i5\pi/6})$	$(0.1e^{i3\pi/7}, 0.8e^{i\pi/7}, 0e^{i0})$

1.2.13 Linguistic valued soft set (LVSS) [153].

An order pair (F^L, E) is said be a LVSS over X if, F^L is a function such that, $F^L : E \to P_L(X)$. Thus, a LVSS (F^L, E) over the set X can written as below,

$$(F^L, E) = \{(e_j, F^L(e_j))|\forall e_j \in E\}$$

where, $F^L(e_j) = \{(x_s, F^L(e_j)(x_s))|\forall e_j \in E, x_s \in X\}$. $F^L(e_j)(x_s)$ is the linguistic rating of an alternative $x_s \in X$ based on a parameter $e_j \in E$.

Example 1.6. Now, consider a linguistic term set as, S=$\{s_{-2}$(very low), s_{-1}(low), s_0(fair), s_1(high), s_2(very high)$\}$. Again let, $X = \{y_1, y_2, y_3\}$ be the three houses as the initial universe and $E = \{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ be the three linguistic valued parameters where, ε_1 stands for 'price', ε_2 stands for 'in green surroundings' and ε_3 stands for 'outlook'.
Now by using the above linguistic variables, description of 'the attractiveness of the three houses over the three considered parameters' has been provided below:
$F^L(\varepsilon_1) = \{(y_1, s_{-2}), (y_2, s_1), (y_3, s_0)\}$, $F^L(\varepsilon_2) = \{(y_1, s_1), (y_2, s_2), (y_3, s_{-1})\}$,
$F^L(\varepsilon_3) = \{(y_1, s_{-2}), (y_2, s_{-2}), (y_3, s_2)\}$.
Then, $(F^L, E) = \{(\varepsilon_1, ((y_1, s_{-2}), (y_2, s_1), (y_3, s_0))), (\varepsilon_2, ((y_1, s_1), (y_2, s_2), (y_3, s_{-1}))),$ $(\varepsilon_3, ((y_1, s_{-2}), (y_2, s_{-2}), (y_3, s_2)))\}$ be a LVSS over X. In Table 1.6, its tabular form has been given.

Table 1.6: LVSS (F^L, E) based on Example 1.6

	ε_1	ε_2	ε_3
y_1	s_{-2}	s_1	s_{-2}
y_2	s_1	s_2	s_{-2}
y_3	s_0	s_{-1}	s_2

1.3 Historical literature review on soft set theory

In the previous literature, there exist several types of works on soft set theory. Some of them are related to the theoretical developments of soft set theory including, algebraic operations, algebraic structures, various algebraic laws, etc. and some of them are related to the uses of soft set theory in different areas like decision-making, supplier selection, medical diagnosis, etc. Now, we have given a brief literature review on soft set theory in several directions.

1.3.1 Literature review on classical soft sets.

In the pioneering article [118], Molodtsov gave the introduction of soft set theory together with some of its applications in different fields such as in game theory, operations research, stability analysis, real analysis, etc. After Molodtsov [118], Maji et al. [107] involved to work on soft set theory. They suggested some classical algebraic operations through soft sets including complement, AND, OR, union, intersection, etc. Moreover, they also utilized these operations for solving decision-making problems. After that, Ali et al. [11] detected some drawbacks of the operations defined by Maji et al. [107]. Then, in order to overcome these drawbacks, they proposed some new operations. Further, Pei & Miao [127] and Sezgin & Atagün [146] referred some new set-theoretic operational laws for soft sets. Since in a soft set, a finite set of alternatives are evaluated with respect to some parameters, so for m alternatives and n parameters, we will get total $m \times n$ number of evaluations. By using this idea, Çağman and Enginoğlu [33] represented a soft set in matrix form and introduced the idea of soft matrix which has been conveniently used in solving real-life decision-making problems. Furthermore, to comparing two items, similarity measure and distance measure are two very significant issues where, similarity measure is in favor of their closeness degree and distance measure is in favor of their deviation degree. In many different fields, like, pattern recognition, medical diagnosis, data meaning, remote sensing, etc., these two notions have been successfully applied for handling problems. Then, Majumdar and Samanta [109] defined similarity measure approach through soft sets and used it in medical science. Kharal [88] and Yang [179] proposed another new approach of similarity measure for soft sets from a different aspect. Meanwhile, researchers have given an emersion of interest on introduction of classical algebraic structures like relation, mapping, group theory, subgroups, normal subgroups, homomorphism, BCK/BCI- algebra, etc. on soft set theory. For more details, reader are referred to the existing articles [16, 21, 83, 110, 133, 147, 188].

1.3.2 Literature review on fuzzy soft sets.

Fuzzy soft set is a very useful generalization of classical soft set theory by integrating soft set with the fuzzy set in such a manner that, each of its associated parameters is in fuzzy nature. In 2002, Maji et al. proposed this novel idea where, the main feature is to express uncertainty through fuzzy notion without introducing a membership function. Therefore, fuzzy soft set has a remarkable impact in solving our day-to day life related problems. Basic set-theoreticoperations and properties on fuzzy soft sets have been given in reference [139]. Then, Çağman and Enginoğlu represented fuzzy soft sets in matrix forms [36] and used it for easily handling decision-making over fuzzy soft sets. Moreover, in the articles [4, 61, 92, 93, 139] some novel methodologies have been proposed to deal with decision-making problems based on fuzzy soft sets. Furthermore, Basu et al. [22], Celik and Yamak [40], Tang [155], Li et al. [99], etc. have used fuzzy soft set theory in solving disease diagnosis decision-making problems. Besides these effective works, similarity measure for fuzzy soft sets [112] has also been introduced and employed in decision-making [98, 148]. In addition, researchers have proposed several important algebraic structures on fuzzy soft sets such as, fuzzy soft relation [73, 188], fuzzy soft lattice [136], fuzzy soft group [20, 102, 140], fuzzy soft semigroup [178], fuzzy soft ideal [178], etc.

1.3.3 Literature review on intuitionistic fuzzy soft sets.

Intuitionistic fuzzy soft set is another extension of soft set where, the evaluation of an alternative over a parameter is in intuitionistic fuzzy membership i.e., in terms of the order pair of membership as well as non membership such that, $membership + non\ membership \leq 1$. In reference [105], Maji et al. provided its mathematical structure. Then, Xu et. [173] studied some basic operations including intersection, complement, union, etc. over the intuitionistic fuzzy soft sets. Yin et al. [182] developed the lattice structure based on intuitionistic fuzzy soft sets. Meanwhile, Karaaslan et al. [86] proposed group theory through intuitionistic fuzzy soft sets. Besides these theoretical aspects, some of its applications on decision-making has been developed by the researchers. For a comprehensive study, readers are advised to read these articles [2, 53, 82].

1.3.4 Literature review on interval type-2 fuzzy soft sets.

Interval type-2 fuzzy set is the mostly used type-2 fuzzy set in many practical fields [38, 79]. Then, Zhang and Zhang [187] initiated the idea of interval type-2 fuzzy soft set where, all the parameters are in interval type-2 fuzzy sense. They also studied a decision-making by using this kind of soft set. Then, Khalil and Hassan [89] investigated this article [187] and demanded some limitations of some propositions proposed by Zhang and Zhang [187] and further offered their modified versions. Latter, Zhang and Zhang [190] analyzed Khalil and Hassan's revised article [89] and detected some limitations and produced their modified versions.

1.3.5 Literature review on linguistic valued soft sets.

In practice, all the information in our day-to-day life not in quantitative form but in qualitative one like low, very low, high, etc. Since, in a soft set, we can take any parameter to define an element or object, then Sun et al. [153] proposed idea of linguistic valued soft set where, objects are defined in terms of linguistic evaluations. Further, Tao [157] initiated the concept of 2-tuple linguistic soft set by integrating 2-tuple linguistic valued set with the soft set. Guan et al. [71] considered intuitionistic fuzzy linguistic information in a soft set and then introduced the notion of intuitionistic fuzzy linguistic soft set. Moreover, Zhao et al. [191] proposed the idea of fuzzy valued linguistic soft set where, the rating of an object or an alternative over a parameter is in linguistic variable and additionally, every linguistic variable has a fuzzy membership grade. In the references [71, 153, 157, 191], researchers have also analyzed the applications of the proposed linguistic valued soft sets in decision-making.

1.3.6 Literature review on complex fuzzy soft sets and complex neutrosophic soft sets.

Complex fuzzy set is a new extension of fuzzy set theory where, every element is defined through complex fuzzy membership. Complex intuitionistic fuzzy set [8], complex neutrosophic set [12], complex vague set, etc. are some extensions of complex fuzzy set. Then, by using these newly proposed uncertain environments in soft set theory, researchers have proposed the notion of complex fuzzy soft set [158], complex intuitionistic fuzzy soft set [94], complex vague soft set [144] and complex neutrosophic soft set [29]. Furthermore, some application fields of these theorems have also been been saved in literature.

1.4 Motivations and objectives of the thesis

In late 19th century, uncertainty has been considered as a cogent plague in science and engineering. Probability theory, fuzzy set theory, linguistic approach, etc. are some of the relevant theorems to conduct uncertainty. Although these ground works have a great utility sometimes, they are disavowed their pertinency. Then, Prof. Molodtsaov [118] proposed a new notion of soft set theory by using parameterization. In soft set theory, introducing of membership function as like fuzzy set theory or else performing enormous number of trials as like probability theory have been omitted. Consequently, it becomes an emergent approach for modelling uncertainty. During literature survey it is observed that, fuzzy soft set theory has a huge utilization in decision-making as well as in group decision-making problems [64] where, a set of finite number of alternatives have been sorted with respect to some fuzzy parameters based on different experts opinions. However, in many real-life related problems, it is necessary to measure the consensus of the opinions of the experts in providing their associated fuzzy soft set, because the experts may come from different environments and they may have different choices. But, in literature, there exist no research on fuzzy soft set based group decision-making where, consensus level of the experts is measured. Then, to address this notion, our first objective is, to propose a group decision-making approach based on fuzzy soft set theory by measuring agreement level or consensus level of the experts.

In 1971, first Rosenfeld proposed the idea of fuzzy group [138] by introducing group theory through fuzzy set. After that, some algebraic structures such as, normal subgroup [117], cyclic group [14], etc. have also been saved in existing conjectures through fuzzy set. Meanwhile, Aygünoğlu and Aygün [20] initiated fuzzy soft group by combining the notion fuzzy group with soft set theory. Further, they also defined the concept of function, homomorphism, normal subgroup, etc. on fuzzy soft sets. But, up to date, no researcher has worked on cyclic group through fuzzy soft set framework. This research gap demands a new research work on cyclic group based on fuzzy soft sets. Therefore, our second objective is, to develop the idea of fuzzy soft cyclic group.

However, considering parameters in qualitative from (linguistic valued) is more appreciable than the quantitative one. Consequently, linguistic valued soft sets [153], 2-tuple linguistic soft sets [156], Uncertain linguistic fuzzy soft sets [157], fuzzy-valued linguistic soft sets [191], etc. have been initiated and used in handling decision-making. Recently, Sun et al. [153] offered a group decision-making for linguistic valued soft set to get a solution from a group decision-making problem. But, we have verified that, in this existing algorithm, weight finding procedure of an expert is not true in general. Then, to overcome this lacuna, we have developed a new approach for solving linguistic valued soft set based multi-criteria group decision-making problems which exceed the limitations of Sun's algorithm. This is the third objective of this proposed thesis.

Trapezoidal fuzzy soft set was proposed by Xiao et al. [170] is based on trapezoidal fuzzy numbers. They also applied it to cover up MCDM problems. But, from the existing literature we do not find any work on trapezoidal intuitionistic fuzzy soft set theory which can be commonly used in real-life related problems. Therefore, our fourth objective is, to propose an approach on trapezoidal intuitionistic fuzzy soft sets.

In 2013, Zhang and Zhang [187] introduced a new hybridization of soft set theory with trapezoidal interval type-2 fuzzy numbers. Mathematically, it is named as trapezoidal interval type-2 fuzzy soft set. In addition, they also developed an algorithm for handling MCDM problems involving multiple experts through the notion of trapezoidal interval type-2 fuzzy soft sets. However, in real-life, the rating of an alternative over a parameter may be changed stochastically with respect to some possible states. Therefore, our fifth objective is, to solve decision-making under stochastic environment by using trapezoidal interval type-2 soft set.

Complex fuzzy soft set theory is a recent developed model where with respect to each of the associated parameters, every alternative has a complex fuzzy membership in terms of amplitude part and phase part by which we can solve the problems where, an additional information has to be expressed with respect to every parameter. So, this approach is more flexible and applicable to solve day-to-day life problems. Therefore, we have motivated to work on complex fuzzy soft set theory. Hence, our sixth objective is, to develop a complex fuzzy soft decision-making and its application in real-life.

Ali et al. [12] proposed the novel concept of complex neutrosophic set where, truth membership, indeterminate membership and false membership play independently in terms of complex-valued. Further, Broumi et al. [29] introduced the notion of complex neutrosophic soft set by combining complex neutrosophic set with soft set. But, this idea is not well established till now. Therefore, our seventh objective is to use complex neutrosophic soft set in solving decision-making problems so that we can solve more complicated real-life related problems.

1.5 Organization of the thesis

In the proposed thesis, some new generalizations of soft set theory have been discussed and applied in different real-life related fields. Besides, some algebraic structures of soft set theory under different uncertain environments have been proposed. Our proposed thesis has been partitioned into the following ten chapters.

- **Chapter 1:** General introduction and organization of the thesis
- **Chapter 2:** Consensus measuring and reaching to consensus threshold in fuzzy soft set based group decision-making by using distance measure
- **Chapter 3:** Some properties of fuzzy soft groups
- **Chapter 4:** A new algorithmic approach to linguistic valued soft multi-criteria group decision-making problems using linguistic scale function
- **Chapter 5:** Generalized trapezoidal intuitionistic fuzzy soft sets in risk analysis
- **Chapter 6:** Trapezoidal interval type-2 fuzzy soft stochastic set and its application in stochastic multi-criteria decision-making
- **Chapter 7:** Application of complex fuzzy soft sets in medical diagnosis system through a similarity measure approach
- **Chapter 8:** A soft set based VIKOR approach for some decision-making problems under complex neutrosophic environment
- **Chapter-9:** Summary and future research works

- **Chapter-10:** Bibliography

Chapter 1: General introduction and organization of the thesis

This chapter contains a general introduction on soft set theory under different uncertain environments along with some historical literature reviews. A brief preliminary studies have been given in this chapter. Moreover, motivations and objectives of this thesis have also been added in this chapter.

Chapter 2: Consensus measuring and reaching to consensus threshold in fuzzy soft set based group decision-making by using distance measure

In the current phase of research, group decision-making through fuzzy soft set theory has become a well-accepted strait to the researchers. However, due to diverse backgrounds of the associated decision makers, it is a very difficult task to them to provide similar types of opinions about an alternative, in spite of that, to complete a transparent decision process and to get a quality solution from the problem, opinions of all the decision makers should be taken into consideration. Such type of situations in a problem demand consensus measuring mechanism for the decision makers. Therefore, in this chapter, we have studied a consensus measuring and reaching approach to consensus threshold in fuzzy soft set based group decision-making. In this regard, a new algorithm has been proposed for solving fuzzy soft set based group decision-making problems by utilizing fuzzy distance and fuzzy soft distance via three main steps: consensus level measuring step, consensus level increasing step and then, best decision solution selection step. After that, we have demonstrated a case study on sustainable supplier selection problem in a textile industry to show the applicability of our proposed approach. A comprehensive comparative discussion has been illustrated from which it has been proven that, our proposed methodology is more logical and effective than the other existing fuzzy soft set based group decision-makings.

Chapter 3: Some properties of fuzzy soft groups

In this chapter, firstly, we have defined the order of an element of a fuzzy soft group. Then, we have initiated the notion of a fuzzy soft cyclic group with respect to a fuzzy soft group over a universal set X. Some related properties and theorems have also been developed in this chapter. Moreover, we have also shown that, intersection of two fuzzy soft cyclic group is a fuzzy soft cyclic group and for the case of union of two fuzzy soft cyclic groups, it has been concluded that, with some certain condition, union of two fuzzy soft cyclic groups is a fuzzy soft cyclic group. We have also discussed some examples to justify the existence of these proposed definitions and theorems.

Chapter 4: A new algorithmic approach to linguistic valued soft multi-criteria group decision-making problems using linguistic scale function

In this chapter, some group decision-making problems based on linguistic valued soft set theory have been studied by using linguistic scale function. Firstly, a new similarity measure for linguistic valued sets has been defined. Then, we have introduced a ranking function for an alternative in a linguistic valued soft set with the help of a linguistic scale function. Additionally, we have also introduced the satisfaction level of an expert in a linguistic valued soft set based group decision-making problem. Finally, by using the above-proposed notions, a stepwise decision-making algorithm has been provided to solve linguistic valued soft set based multi-criteria group decision-making problems. Then, we have clarified this methodology by a plant location selection problem of a manufacturing company. Besides, a comparative analysis has been discussed to prove the feasibility and efficiency of our approach.

Chapter 5: Generalized trapezoidal intuitionistic fuzzy soft sets in risk analysis

In many real-life related fields, specially in medial science, some times, qualitative information is more adequate than the quantitative value. This chapter represents a mathematical approach with linguistic information intuitively by using soft set theory under generalized trapezoidal intuitionistic fuzzy environment to recognize a patient whether he/she is a diabetic or not. Therefore, firstly, we have proposed the notion of generalized trapezoidal intuitionistic fuzzy soft set by combining soft set theory with generalized trapezoidal intuitionistic fuzzy set so that, each of the evaluations is in of generalized trapezoidal intuitionistic fuzzy number. Then, we have developed some classical operations such as, union, ‘complement’, ‘intersection’, etc. on generalized trapezoidal intuitionistic fuzzy soft sets. After that, a distance measure approach have been defined for generalized trapezoidal intuitionistic fuzzy soft sets. Finally, a new decision-making approach has been proposed to solve a generalized trapezoidal intuitionistic fuzzy soft set based decision-making problems. As a applicability of our proposed approach, a diabetic patient recognition problem has been illustrated and solved by using our proposed approach.

Chapter 6: Trapezoidal interval type-2 fuzzy soft stochastic set and its application in stochastic multi-criteria

decision-making

In this chapter, we have dealt with trapezoidal interval type-2 fuzzy soft set based decision-making problems, where, the evaluation of an alternative over a parameter changes stochastically. In this regard, at first, we have defined the concept of trapezoidal interval type-2 fuzzy soft stochastic set (TIT2FSSS). Then, the definition of expected trapezoidal interval type-2 fuzzy soft set has been proposed. After that, we have developed a decision-making methodology to handle trapezoidal interval type-2 fuzzy soft stochastic set based stochastic multi-criteria decision-making problems. In addition, to obtain the weights of the parameters where, the weights of the parameters are partially known, we have proposed a process by using signed distance measurement. Finally, a real-life example has been discussed to illustrate the efficiency and feasibility of our proposed approach.

Chapter 7: Application of complex fuzzy soft sets in medical diagnosis system through a similarity measure approach

Many of today's research theme are applying machine vision and artificial intelligence in medical disease diagnosis system. Besides, uncertainty is a salient feature in medical practice to be dealt very carefully for providing a patient's proper health status. Recently, Molodtsov's soft set theory is used in a glaring manner in disease diagnosis decision-making under fuzzy uncertain environment. Now, this chapter focuses on multi-expert decision-making by using soft set theory under complex fuzzy environment to detect accurate disease of a patient having various types of symptoms with different inclusion rate at different time. Firstly, a ratio similarity measure approach for complex fuzzy soft sets has been introduced where, the detailed proof of satisfying the axiomatic properties has also been given. Then, we have defined complex fuzzy soft weighted geometric mean aggregation operator to combine multiple complex fuzzy soft sets provided by multiple experts into a single complex fuzzy soft set. Finally, an algorithmic approach has been developed by combining these above-proposed ideas for solving complex fuzzy soft set based multi-expert decision-making problems. After that, a case study on disease diagnosis of a patient has been illustrated based on three experts opinions. Moreover, a comparative analysis has been studied to verify the validity and effectiveness of our proposed approach. In addition, we have discussed the sensitivity analysis of our approach which supports its stability.

Chapter 8: A soft set based VIKOR approach for some decision-making problems under complex neutrosophic environment

In this chapter, a decision-making algorithm has been developed to deal with complex neutrosophic soft set based problems. At first, we have defined some basic operations including various types of unions, intersections, aggregations, on complex neutrosophic sets as well as on complex neutrosophic soft sets. Then, we have introduced a new definition of score function for a complex neutrosophic number to convert a complex neutrosophic number into some real number in the interval $[0, 1]$. After that, a complex neutrosophic soft set based VIKOR approach has been offered to obtain a compromise optimal solution from a complex neutrosophic soft set based decision-making problem. Some real life based problems have been solved by our approach which show the applicability of our proposed approach.

Chapter 9: Summary and future research works

In this chapter, summary of the proposed thesis and some future research works of this thesis have been discussed.

Chapter 10: Bibliography

In this chapter, some references have been added.