M.Sc. 1st Semester Examination, 2024

APPLIED MATHEMATICS

(*Classical Mechanics and Non-linear Dynamics*)

PAPER – MTM-105

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

1. Answer any *four* questions : 2×4

(*a*) Define the Lagrangian and Hamiltonian of a dynamical system. Compare these two functions.

(*b*) What do you mean by non-inertial frame? Give an example of a non-inertial frame.

(*c*) Is Poisson bracket commutative? Justify your answer.

(*d*) State and prove the conservation law of angular momentum.

(*e*) Define cyclic coordinates. State when Routhian equations of motion are useful.

(*f*) For any dynamical quantities, X, Y, Z, prove that $[X+Y, Z] = [X, Z] + [Y, Z]$.

GROUP – B

2. Answer any *four* questions : 4×4

(*a*) Define constant of motion. Show that the Poison bracket of two constants of motion is also constant of motion.

(*b*) Solve the Euler's dynamical equations

$$A\dot{w}_1 - (B - C)w_2 w_3 = 0,\ B\dot{w}_2 - (C - A)w_3 w_1 = 0,$$

$$C\dot{w}_3 - (A - B)w_1 w_2 = 0, \quad \text{when } A = B.$$

(*c*) What is the effect of the Coriolis force on a particle falling freely under the action of gravity ?

(*d*) Derive the differential equations of the lines of propagation of light in an optically non-homogeneous medium with the speed of light $c(x, y, z)$. Also, discuss the case when c is constant.

(*e*) The Lagrangian for a system of one degree of freedom can be written as

$$L = \frac{m}{2}\left(\dot{q}\sin wt + \dot{q}\, q w \sin 2wt + q^2 w^2\right).$$

Determine the corresponding Hamiltonian.

(*f*) Show that the following transformation is canonical

$$Q_1 = \frac{1}{\sqrt{2}}\left(q_1 + \frac{p_2}{mw}\right), \quad P_1 = \frac{1}{\sqrt{2}}(p_1 - mwq_2)$$

$$Q_2 = \frac{1}{\sqrt{2}}(q_1 - p_2 mw), \quad P_2 = \frac{1}{\sqrt{2}}(p_1 + mwq_2).$$

GROUP – C

3. Answer any *two* questions : 8×2

(*a*) Deduce Lagrange equations of motion for unconnected holonomic and non-conservative force.

(*b*) Derive the Lorentz transformation equations.

(*c*) State Hamilton's principle. Deduce it from D'Alembert's principle.

(*d*) Derive the Euler-Lagrange differential equations for multiple dependent variables.

Also, find such equations for the following problem : 5 + 3

$$J = \int_{t_0}^{t_1} \frac{m}{2}[(\dot{x}^2 + \dot{y}^2) - mgy]dt.$$

[Internal Assessment — 10 Marks]