M.Sc. 1st Semester Examination, 2024

APPLIED MATHEMATICS

(*Graph Theory*)

PAPER – MTM-106

Full Marks : 25

Time : 1 hour

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Illustrate the answers wherever necessary

1. Answer any *two* questions : 2×2

(*a*) Prove that a tree with two or more vertices has at least two pendant vertices.

(*b*) Draw the digraph G corresponding to adjacency matrix A :

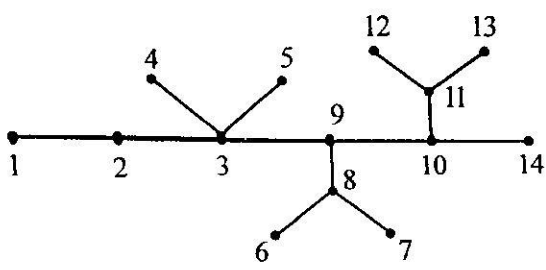
$$A = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

(*c*) Write down the properties of binary trees.

(*d*) Show that a simple connected graph with n vertices has at most $n(n-1)/2$ edges.

2. Answer any *two* questions : 4×2

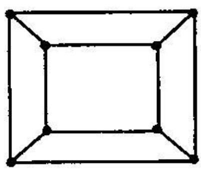
(*a*) Define the radius, diameter and centre of a graph. Find the radius, diameter and centre for the following graph.



(*b*) Prove that if T is a tree with n vertices then it has precisely $(n\text{-}1)$ edges.

(*c*) Show that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges.

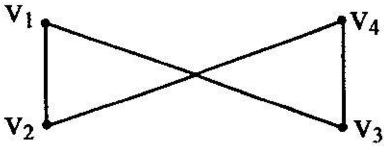
(*d*) Define the Eulerian graph and Hamiltonian graph. Show that the following graph is Hamiltonian but not Eulerian.



3. Answer any *one* question : 8×1

(*a*) (*i*) State and prove Euler's theorem for a connected planar graph. 4

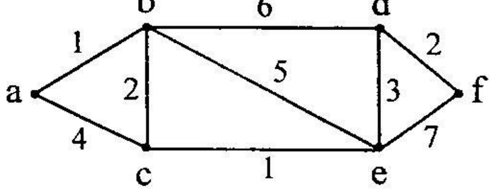
(*ii*) Consider the graph shown in figure, find the number of walks of length three from V_2 to V_4 and also check the connectedness of the graph. 4



(*b*) (*i*) Prove that the chromatic polynomial of any cycle C_n of length n is

$$p_n(\lambda) = (\lambda - 1)^n + (-1)^n(\lambda - 1).$$ 3

(*ii*) Apply Dijkstra's algorithm to the graph given below, find the shortest path from a to f 5



[Internal Assessment — 5 Marks]