# M.Sc. 3rd Semester Examination, 2024

## MATHEMATICS

( *Transforms and Integral Equations* )

PAPER – MTM-302

*Full Marks* : 50

*Time* : 2 hours

Answer **all** questions

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

1. Answer any *four* questions from the following : 2 × 4

   (*a*) If $\overline{G}(k,l)$ be the two-dimensional Fourier transform of a function $G(x,y)$, then what is the Fourier inversion formula to get $G(x,y)$ from $\overline{G}(k,l)$.

( *Turn Over* )

(*b*) Find the Laplace transform of $H(t-a)$, where $H(t)$ denotes the Heaviside unit step function.

(*c*) Who first coined the term wavelet ? Define 'mother wavelet' and explain the utility of it.

(*d*) Define infinite Fourier transform and state the conditions of existence of the transform.

(*e*) Define Fredholm integral equation of second kind with an example.

(*f*) Show that $u(x)=x$ is a solution of

$$u(x)=\frac{5}{6}x-\frac{1}{9}+\frac{1}{3}\int_0^1 (x+t)\,u(t)\,dt.$$

**2.** Answer any *four* questions from the following : $4 \times 4$

(*a*) Show that if a function $f(x)$ defined on

$(-\infty, \infty)$ and its Fourier transform $F(\zeta)$ are both real, then $f(x)$ is even. Also show that if $f(x)$ is real and its Fourier transform $F(\zeta)$ is purely imaginary, then $f(x)$ is odd.

(*b*) Define wavelet transform. Write down the main advantages of wavelet theory. Compare the wavelet transform with Fourier transform.

(*c*) Solve the Abel integral equation

$$\int_0^x \frac{f(\xi)}{(x-\xi)^{1/2}}\, d\xi = x.$$

(*d*) Show that two eigen functions corresponding to distinct eigen values of a homogeneous Fredholm integral equation of second kind with symmetric kernel are orthogonal.



( *Turn Over* )

(*e*) Solve the integral equation :

$$y(x) = f(x) + \lambda \int_0^1 (x+t) y(t) dt$$

and find the eigen values.

(*f*) Prove that $L\left(\frac{\sin t}{t}\right) = \tan^{-1}\frac{1}{p}$, where $p$ is the Laplace transform parameter.

3. Answer any *two* questions from the following : 8 × 2

(*a*) Find the solution of the following problem of free vibration of a stretched string of an infinite length :

$$\frac{\partial^2 u}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0. \quad -\infty < x < \infty,$$

subject to $u(x, 0) = f(x)$,

$$\frac{\partial}{\partial t} u(x, 0) = g(x),\ u \text{ and } \frac{\partial u}{\partial x} \text{ both}$$

vanish as $|x| \to \infty$.

(*b*) Solve

$$u(x) = 1 + \frac{1}{5}\int_0^{\frac{\pi}{3}} \sec x \tan x \ u(t)dt$$

using the method of successive substitution.

(*c*) (*i*) With help of the resolvent kernel, find the solution of the integral equation

$$y(x) = 1 + x^2 + \int_0^x \left(\frac{1+x^2}{1+t^2}\right) y(t)dt.$$

(*ii*) Discuss the solution procedure of homogeneous procedure of homogeneous Fredholm integral equation of the second kind with degenerate kernel. 4 + 4

(*d*) State and prve the Perseval theorem on Fourier transform. Find the Fourier transform of

$$f(x) = (1 - |x|)\, H(1 - |x|),$$

where $H(x)$ is the Heaviside unit step function, hence show that

$$\int_0^\infty \left(\frac{\sin x}{x}\right)^4 = \pi/3.$$

**[ Internal Assessment — 10 Marks ]**