# M.Sc. 3rd Semester Examination, 2024

## APPLIED MATHEMATICS

PAPER – MTM-303

*Full Marks* : 50

*Time* : 2 hours

Answer **all** questions

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

## PAPER – MTM-303(Unit-1)

( *Stochastic Process and Regression* )

**[ Marks : 20 ]**

1. Answer any *two* questions : $2 \times 2$

   (*a*) State the problem of the reflecting barrier and write the transition matrix for it.

(*b*) Define multiple correlation coefficient and partial correlation coefficient.

(*c*) Discuss the concept of higher transition probabilities in Markov chains.

(*d*) How can you identify ergodic states in a Markov chain ?

2. Answer any *two* questions : $4 \times 2$

(*a*) Define the communicate relation. Prove that communicate relation is transitive.

(*b*) For any state $j$ and $k$, prove that

$$P_{jk}^{(n)} = \sum_{r=0}^{n} f_{jk}^{(r)} p_{kk}^{(n-r)}, \quad n \geq 1$$

with $p_{kk}^{(0)} = 1, f_{jk}^{(0)} = 0, f_{jk}^{(1)} = p_{jk}$.

(*c*) Consider the Markov Chain with the following transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Test whether the states are periodic and persistent.

(*d*) Define Markov chain. Explain, with suitable example, the classification of states in a Markov chain.

**3.** Answer any *one* question : $8 \times 1$

(*a*) Starting from the differential-difference equation for the birth and death process, find the probability generating function when the rate of birth and death are respectively $n\lambda$ and $n\mu$, where $n$ is the population size at any time $t$. Assume that the initial propulation size is $i$.

(*b*) (*i*) What is power density Spectrum ?

(*ii*) An ergodic process is known to have an auto-correlation function of the form.

$$R_{xx}(\tau) = 1-|\tau|, \ |\tau| \leq 1$$
$$= 0, \qquad |\tau| > 1$$

Find the spectral analysis. 4 + 4

**PAPER — MTM-303(Unit-2)**

(*Cryptograpgy*)

**[ Marks : 20 ]**

**4.** Answer any *two* of the following questions : $2 \times 2$

(*a*) Define the terms 'secret key' and 'cryptology'.

(*b*) Write down two important requirements for secured use of single-key encryption.

(*c*) Write down a short note on "stream ciphers".

(*d*) Explain the term 'public-key infrastructure'.

**5.** Answer any *two* of the following questions : $4 \times 2$

(*a*) Explain the concepts 'Hypertext transfer protocol' and 'public-key certificate'.

(*b*) Write down the differences between symmetric encryption and public-key encryption for security and performance.

(*c*) Write down the algorithm of Data encryption standard and illustrate it graphically.

(*d*) Write down a short note on "Vernam ciphers".

6. Answer any *one* of the following question : $8 \times 1$

(*a*) (*i*) Write down the algorithm of Caesar cipher for encryption and decryption. 2

(*ii*) Write down the algorithm of Hill cipher and illustrate it using an example. 3 + 3

(*b*) (*i*) Describe Feistel cipher structure and draw its graphical representation to encrypt the data. 5

(*ii*) Write down the essential steps for the public-key cryptosystem and illustrate it graphically. 3

**[ Internal Assessment — 10 Marks ]**