M. Sc. 3rd Semester Examination, 2024

APPLIED MATHEMATICS

(Discrete Mathematics)

PAPER — C-MTM-304 (CBCS)

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any **four** questions : 2 × 4

   (*a*) Define path, cycle and complete graphs and give an example of each of these graphs.

(2)

(b) Define the degree sequence of a graph. Draw the graphs whose degree sequences are 4, 4, 4, 4, 4 and 3, 3, 2, 2, 2.

(c) Define the complete lattice with an example.

(d) What is a meet semi-lattice ? Give an example.

(e) Find a closed form for the generating function of the following sequence :
1, 1, 0, 1, 1, 1, 1, 1,.........

(f) Write down the duality principle of Boolean algebra. Find the dual of the Boolean expression : $xy(y + y'z) + x'z$

2. Answer any *four* questions : 4 × 4

(a) Define the Eulerian graph and Hamiltonian graph. Draw two graphs one of which is Hamiltonian and another is not Hamiltonian. Is complete bipartite graph Hamiltonian or Eulerian ? Justify your answer by an example.

(*Continued*)

(b) Show that the following graphs are Hamiltonian but not Eulerian.
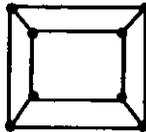
(c) Define the Poset and Toset with examples. Define modular lattice and distributive lattice. Cite example for each case.

(d) (i) Prove that a chain is a distributive lattice.

(ii) In any lattice L, prove that $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \leq (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$, $\forall\ a, b, c \in L$. 2+2

(e) In the Boolean algebra (B, +, ., '), a Boolean function is defined by $f(x, y, z) = xy + yz + zx$. Find the disjunctive normal form (DNF) and conjunctive normal form (CNF) of $f(x,y,z)$.

(f) Using mathematical induction, show that

$$3 + 33 + 333 + \ldots\ldots + 33\ldots\ldots3 = (10^{n+1} - 9n - 10)/27.$$

3. Answer any **two** questions : 8×2

(a) Define the radius, diameter and centre of a graph. Find the radius, diameter and centre for the following graph.
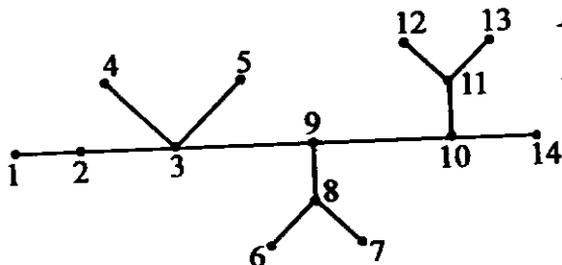
(b) (i) Define finite-state machine with example. 2

(ii) Let M be the finite-state machine with state table appearing in the following.

(*Continued*)

(*I*) Find the input set Σ, the state set S, the output set O and the initial state of M.

(*II*) Draw the state diagram of M.

(*III*)Find the output string for the input string aabbcc.   2 × 3

S \ Σ	f			g		
	a	b	c	a	b	c
s_0	s_0	s_3	s_2	0	1	1
s_1	s_1	s_1	s_3	0	0	1
s_2	s_1	s_2	s_3	1	1	0
s_3	s_2	s_3	s_0	1	0	1

(*c*) (*i*) Determine the generating function of the following sequences :   4

$$a_r = 5^r + (-1)^r 3^r + 8^r + {}^3 c_r$$

(*ii*) Use generating function to solve the recurrence relation :   4

$$a_n = 3a_{n-1} + 2 \text{ taking } a_0 = 1$$

(*d*) State the principle of inclusion-exclusion. Use the principle of inclusion-exclusion, find the number of primes less than 100.

2 + 6

[Internal Assessment — 10 Marks]