M.Sc. 3rd Semester Examination, 2024

APPLIED MATHEMATICS

PAPER – MATH-306(A, B & C)

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER : MTM-306(A)

(*Operational Research Modeling-I*)

1. Answer any *four* questions : 2×4

   (*a*) What are the critical paths and critical activities in network analysis ?

(*b*) What is simulation ? Write down the limitations of the simulation.

(*c*) What are the differences between PERT and CPM ?

(*d*) Define traffic density in a queuing model. Discuss the situations when traffic density is equal to 1, greater than 1 and less than 1.

(*e*) Define the following terms : Replenishment and Shortage cost.

(*f*) Why do we need to control the inventory ?

2. Answer any *four* questions of the following : 4×4

(*a*) Write the different types of service disciplines used in queuing theory.

(*b*) Discuss the common errors that occur in a network construction.

(*c*) The following table given data for a project :

Activity	Time (days)
1-2	6
1-3	4
2-4	5
2-5	3
3-4	6
4-6	8
5-6	4
6-7	3

Draw the network and the minimum completion time of the project.

(*d*) Derive the EOQ formula of manufacturing inventory model without shortages, finite replenishment rate and zero lead time system.

(*e*) A machine owner finds from his past record that the costs per year of main

taining a machine whose purchase price is Rs. 6000 are given in the following table :

Year:	1	2	3	4	5	6	7	8
Resale price:	3000	1500	750	375	200	200	200	200
Maintenance cost:	1000	1200	1400	1800	2300	2800	3400	4000

Find the most economic replacement age.

(f) A baking company sells cake by kg. It makes a profit of Rs. 0.50 per kg on every kg of cake sold it is baked. It dispossess of all cakes not sold on the date it is baked at a loss of 15 paisa per kg. If demand is known to be uniform distribution between 2000 kg to 3000 kg. Determine the optimum daily amount baked.

(*Continued*)

3. Answer any *two* questions : 8×2

(*a*) A project consists of eight activities with the following relevant information.

Activity	Time estimates (Week)			Predecessor
	t_0	t_m	t_p	
A	1	1	7	None
B	2	4	7	None
C	2	2	8	None
D	1	1	1	A
E	2	5	14	B
F	2	5	8	C
G	3	6	15	D, E
H	1	2	3	F, G

(*i*) Draw the network and find the expected project completion time.

(*ii*) What is the probability of completing the project 4 weeks before the expected completion time ?

(*b*) A firm has a single channel service station with the following arrival and service time probability distributions :

Inter arrival time (minutes)	Probability	Service Time (Minutes)	Probability
10	0.10	5	0.08
15	0.25	10	0.14
20	0.30	15	0.18
25	0.25	20	0.24
30	0.10	25	0.22
		30	0.14

The customer's arrival at the service station is a random phenomenon and the time between the arrivals varies from 10 minutes to 30 minutes. The service time varies from 5 minutes to 30 minutes. The queuing process begins at 10 A.M. and proceeds for 15 arrivals. An arrival goes to the service facility immediately. If it is free. Otherwise, it will wait in a queue. The queue discipline is first-come-first served. If the attendant's wages are Rs. 10

per hour and the customer's waiting time costs Rs. 15 per hour, then would it be an economical proposition to engage a second attendant ? Answer using the Monte Carlo simulation technique.

(*c*) A workshop produces three machine parts A, B, C and the total storage space availbale is 640 sq. meters. Obtain the optimal lot-size for each item from the following data :

Item	Ordering Cost (Rs.)	Demand, (units per year)	Cost per unit (Rs.)	Storage area required per unit (sq. meter.)
A	100	5000	10	0.60
B	75	10000	5	0.45
C	200	2000	25	0.80

Where the carrying charge on each item is 20% of unit cost.

(*d*) A research team is raise to a strength of 50 chemists and them to remain at that level. The wastage of recruits depends on their length of service which is as follows :

Year	1	2	3	4	5	6	7	8	9	10
Total % who have left upon the end of year	5	36	56	63	68	73	79	87	97	100

What is the recruitment per year necessary to maintain the required strength ? There are 8 senior posts for which the length of service is the main criteria. What is the average length of service after which new entrant expects promotion to one of the post ?

[Internal Assessment — 10 Marks]

PAPER - MTM-306(B)

(*Dynamical Oceanology : Advanced Wave Hydrodynamics*)

1. Answer any *four* questions from the following : 2×4

   (*a*) Define stationary wave with an example.

   (*b*) Write down the kinematical condition of the free surface flow.

   (*c*) Determine the expression of group velocity for the wave length λ.

   (*d*) Write down the conditions for shallow water, intermediate and deep water waves.

   (*e*) Determine the speed of propagation for the steady motion of progressive waves of an interface when the liquids are at rest.

(*f*) Write down a short note on "Sverdrup waves".

2. Answer any *four* questions from the following: 4×4

(*a*) Derive an expression for speed of propagation of a progressive wave in the surface of a channel of finite depth. Hence, show that for large wave length the speed of propagation tends to $\sqrt{gh}$, where h is the depth of the canal.

(*b*) Define capillary waves and prove that the relation for surface tension (T) of capillary waves is

$$\frac{\partial^2 \phi}{\partial t^2} - g\frac{\partial \psi}{\partial x} + \frac{T}{\rho}\frac{\partial^3 \psi}{\partial x^3} = 0$$

where symbols have usual meaning.

(*c*) Let a shallow be filled with oil and water, the depth of water be k and its density σ and the depth of oil h, its density ρ, show that

$$\frac{v^2}{g}=\frac{1}{2}(h+k)+\frac{1}{2}\left\{(h-k)^2+4hk\frac{\rho}{\sigma}\right\}^{1/2}$$

if g be the gravity and v be the velocity propagation of long waves.

(*d*) Define group velocity. Prove that for waves on deep water the group velocity is half of the wave velocity while for shallow water group velocity is equal to the wave velocity.

(*e*) Determine the stream-wise velocity expression for linear waves in the absence of rotation and also display the bell-shapped surface elevation with typical width L.

(*f*) Derive the speed of propagation equation due to effect of capillary on surface waves at an interface. Hence, show that the relation $\frac{c^2}{c_0^2}=\frac{1}{2}\left(\frac{\lambda}{\lambda_0}+\frac{\lambda_0}{\lambda}\right)$

for the liquid is at rest and there depth is higher where symbols have their usual meaning.

3. Answer any *two* questions from the following : 8×2

(*a*) (*i*) Derive depth-averaged momentum equations for shallow water theory.

(*ii*) Define "Rossby radius" and write down its value for deep Ocean whose depth is 4000m. $6 + 2$

(*b*) (*i*) Prove that the total energy of stationary wave is $\frac{1}{4}\rho g a^2 \lambda$ where a, λ is the wave amplitude and wave length respectively.

(*ii*) Prove that the path of the particle is a straight line for the stationary wave on deep water. $5 + 3$

(*c*) Derive Klein-Gordon equation for long surface wave and hence, prove that geostrophic velocity in *y*-direction is given by

$$\bar{v} = \frac{gh}{2C_0}\exp(-|x|/a).$$

where symbols have their usual meaning. 4 + 4

(*d*) Prove that the velocity of propagation is

$$c^2 = \frac{g}{m} \text{ or } c^2 = \frac{g(\rho - \rho')}{m(\rho \coth mh' + \rho')}$$

for the steady motion of progressive waves at an interface when the upper surface is free. Hence, find out the ratio of the wave amplitudue relation to explain the abnormal resistance which experienced by ships where there is a layer of fresh water over a salt. 6 + 2

[Internal Assessment - 10 Marks]

PAPER - MTM-306(C)

(*Computational Fluid Dynamics*)

1. Answer any *four* questions : 2×4

(*a*) Write any four points for each advantage and disadvantages for the use of CFD.

(*b*) What do you mean by grid independent study in the field of computational fluid dynamics ? Also show graphically in the plane : Error versus number of grid.

(*c*) What is convergence, stability and consistency in CFD ? Show the conceptual relationship between them.

(*d*) A generalization fo discretization of one-dimensional unsteady heat conduction equation (with thermal conductivity α) can be obtained by writing

$$\frac{\Delta T_j^{n+1}}{\Delta t} - \alpha[(1-\beta)L_{xx}T_j^n + \beta L_{xx}T_j^{n+1}] = 0$$

Symbols have their usual meaning. Find the value of β for which the above scheme becomes FTCS, Crank-Nicolson and fully implicit.

(*e*) Sketch the control volume for the discretization of the continuity equation while using a staggered grid arrangement, and hence apply finite volume method.

(*f*) What are the advantages of the finite volume method over the finite difference method ?

2. Answer any *four* questions : 4×4

(*a*) Roughly the truncation error E can be written as $E = A\left(\Delta x^k\right)$, with symbol have their own meaning. By plotting on a log-scale, discuss the convergence rates of five-points and three-points symmetry formulas for first order derivative. Also

discuss when do you need to apply higher or lower order formulas for derivative on a non-uniform grid.

(*b*) (*i*) Draw the (j,k)th control volume for irregular grid. Apply the finite volume method on the following equation for the aforesaid control volume

$$\frac{\partial q}{\partial t}+\frac{\partial F}{\partial x}+\frac{\partial G}{\partial y}=0\cdots(1)$$

where the symbols have their usual meaning.

(*ii*) If the global grid is uniform and coincides with lines of constant x and y, then show that the above discretisation coincides with a centered difference representation for the spatial terms of the above original differential equation (1).

(*c*) Derive the composite solution of Richardson extrapolation for FTCS scheme so that the solution error is of $0(\Delta x^4)$ on a sufficiently refined grid.

(*d*) Consider the Sommerfeld radiation condition at the outlet boundary :

$$\frac{\partial \phi}{\partial t} + u_c \frac{\partial \phi}{\partial x} = 0,$$

where φ is the x-velocity component and u_c is the constant local wave speed. Discritise the above equation using the 3-points backward formula for both the derivatives at nth time step over the (N, j)th control volume of staggered grid arrangement.

(*e*) Write the Thomas algorithm for solving a tri-diagonal linear system by writting the general formulas for the modified element of upper triangular matrix and unknown variables.

(*f*) Derive finite volume method for solving one-dimensional steady-state diffusion equation with a source term. Also, sketch the schematic diagram for this scheme.

3. Answer any *two* questions : 8×2

(*a*) (*i*) Write the non-dimensional form of *y*-component Navier-Stokes equations for laminar two-dimensional incompressible fluid flow and then convert it to its conservation form.

(*ii*) Draw the control volume for v-velocity.

(*iii*) Hence apply the finite volume method to the above y-momentum equation for space derivative and three-points backward finite difference formula for time derivative to get the discretise equation in terms of fluxes only.

$2 + 2 + 4$

(*b*) (*i*) Discretize the one dimensional transport equation

$$\frac{\partial T}{\partial t} + a\frac{\partial T}{\partial x} = \alpha\frac{\partial^2 T}{\partial x^2},$$

where a and α are constants, using Crank-Nicolson scheme and hence write the algebraic expression in a matrix form for the case of Neumann boundary conditions.

(*ii*) Write the expression for the First Order Upwind Scheme (FOU) and Second Order Upwind Scheme (SOU). 6 + 2

(*c*) With the help of von Neumann analysis, derive the condition at which the FTCS scheme for $\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}$ is stable.

(*d*) Compute the steady-state temperature distribution from the governing equation

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + q = 0$$

in a large plate of thickness $L = 2$ cm with constant thermal conductivity $k = 0.5$ W/m.K and uniform heat generation $q = 1000\ kW/m^3$. The faces A and B in the x-direction are at temperatures of 100°C and 200°C, respectively. Assuming that y-and z-directions are so large that temperature gradients are significant in the x-direction only. Consider a unit area in the yz-plane, and the five control volumes, giving $\Delta x = 0.004\ m$.

[Internal Assessment — 10 Marks]