M. Sc. 3rd Semester Examination, 2024

APPLIED MATHEMATICS

PAPER – MATH-305 (A & B)

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER – MATH-305(A)

(*Advanced Optimization*)

1. Answer any *four* questions of the following : 2×4

   (*a*) What are the sufficient conditions of occurence of maximum and minimum of a quadratic programming problem ?

(*b*) Why do we need to study the integer programming problem ?

(*c*) Explain different types of achievements in goal programming problem.

(*d*) Explain the deletion of an existing variable from the optimal table of an LPP.

(*e*) State the Bellman's principle of optimality.

(*f*) Define : Unimodal function and Interval of uncertainty.

2. Answer any *four* questions of the following : 4×4

(*a*) Following is the optimal table of an LPP

		C_j	7	9	0	0
C_B	B	x_B	y_1	y_2	y_3	y_4
9	x_2	$\frac{7}{2}$	0	1	$\frac{7}{22}$	$\frac{1}{22}$
7	x_1	$\frac{9}{2}$	1	1	$-\frac{1}{22}$	$\frac{3}{22}$
$z_j - c_j$			0	0	$\frac{28}{11}$	$\frac{15}{11}$

(*Continued*)

Find range of discrete changes of c_1 and c_2 such that the optimal solution does not alter.

(*b*) A firm produces two products A and B. Each product must be processed through two departments namely 1 and 2. Department 1 has 30 hours of production capacity per day, and department 2 has 60 hours. Each unit of product A requires 2 hours in department 1 and 6 hours in department 2. Each unit of product B requires 3 hours in department 1 and 4 hours in department 2. Management has established the following goals it would like to achieve in determining the daily product mix :

P_1 : Producing at least 7 units of product B.

P_2 : Producing at least 8 units of product A.

Formulate the above goal programming problem

(*c*) Explain the basic characteristics of dynamic programming problem.

(*d*) Derive the Kuhn-Tucher conditions for the following QPP.

$$\begin{aligned} \text{Maximize} \quad & Z = 2x_1 + x_2 - x_1^2 \\ & 2x_1 + 3x_2 \le 6 \\ \text{Subject to} \quad & 2x_1 + x_2 \le 4 \\ \text{and} \quad & x_1, x_2 \ge 0 \end{aligned}$$

(*e*) Maximize $f(x) = \begin{cases} \frac{2x}{3}; & x \le 3 \\ 5 - x; & x > 3 \end{cases}$ in the interval [1,5] by Fibonacci method upto five experiments.

(*f*) Derive the expression of Gomory's constraint for the constraint corresponding to non-integer value in the optimal table of an LPP.

(*Continued*)

3. Answer any *two* questions of the following : 8×2

(*a*) Solve the following IPP using branch-and-bound method :

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 2x_2 \\ \text{Subject to} \quad & 5x_1 + 3x_2 \leq 8 \\ & x_1 + 2x_2 \leq 4 \\ \text{and} \quad & x_1, x_2 \geq 0 \text{ are integers.} \end{aligned}$$

(*c*) Solve the following QPP by Beale's method :

$$\begin{aligned} \text{Maximize} \quad & z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 - 4x_1x_2 \\ \text{Subject to} \quad & x_1 + 2x_2 + x_3 = 10 \\ & x_1 + x_2 + x_4 = 9 \\ \text{and} \quad & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

(*d*) Solve the following LPP by the technique of dynamic programming problem :

$$\begin{aligned} \text{Maximize} \quad & z = 2x_1 + 2x_2 \\ \text{Subject to} \quad & 3x_1 + 2x_2 \leq 6 \\ & x_1 + 4x_2 \leq 4 \\ \text{and} \quad & x_1, x_2 \geq 0 \end{aligned}$$

(*e*) Use modified simplex method to solve the following goal programming problem :

$$\begin{aligned} \text{Minimize} \quad & z = P_1 d_1^- + P_2(2d_2^- + d_3^-) + P_3 d_1^+ \\ \text{Subject to} \quad & x_1 + x_2 + d_1^- - d_1^+ = 600 \\ & x_1 + d_2^- = 250 \\ & x_2 + d_3^- = 300 \\ \text{and} \quad & x_1, x_2, d_1^-, d_2^-, d_3^-, d_1^+ \geq 0 \end{aligned}$$

[Internal Assessment — 10 Marks]

PAPER – MTM-305(B)

(Dynamical Meteorology-I)

1. Attempt any *four* questions from the following : 2×4

(*a*) Find the dry adiabatic lapse rate in the atmosphere.

(*b*) How be the mixing ration of the air in the atmosphere calculated using Psychrometer.

(*c*) Show that the adiabatic process is more steeper than the isothermal process.

(*d*) Prove that dry air is heavier than moist air at the same pressure and temperature.

(*e*) What is isochoric process ? Find the work done of an air parcel under isothermal process.

(*f*) What is aerological diagram ? Mention all criteria to be an aerological diagram.

2. Attempt any *four* questions from the following : 4×4

(*a*) Show that the height of the homogeneous atmosphere depends entirely on the temperature at the bottom. Also prove that the pressure at the top of the homogeneous isothermal atmosphere is equal to 1/e times that at the sea level.

(*b*) Prove that if the temperature in the atmosphere falls uniformly with the height ascended, the height (z) of a station above sea level is given by $z = a\left\{1 - \left(\frac{h}{h_0}\right)^m\right\}$, where h and h_0 are the readings of the varometer at the station and at sea level respectively. Also determine a and m which are constants.

(*c*) Discuss different types of humidity variables to measure the amount of water vapor in the atmosphere. Also find the molecular weight of the dry air in the atmosphere.

(*d*) Derive the adiabatic lapse rate for moist saturated air in the atmosphere.

(*e*) Derive the hypsometric equation in the atmosphere.

(*f*) Derive the poisson equation of an air parcel in the atmosphere.

3. Attempt any *two* questions from the following : 8×2

(*a*) Derive the change of lapse rate of temperature when an air layer in the atmosphere ascents or descent dry adiabatically in terms of depth of the layer and pressure changes of top and bottom of the layer.

(*b*) Discuss the adiabatic ascent of an air parcel in the atmosphere. Hence derive the temperature relation at which the air parcel can reach at saturation and then estimate the height at which saturation is attend.

(*c*) Discuss vertical stability analysis of an air parcel in the atmosphere by Parcel Method.

(*d*) What is pseudo-adiabatic process in the atmosphere ? Discuss the equivalent potential temperature and moist static energy in the atmosphere.

[Internal Assessment — 10 Marks]
