# M.Sc. 3rd Semester Examination, 2024

## PHYSICS

PAPER – PHS-301.1 & 301.2

*Full Marks* : 50

*Time* : 2 hours

Answer **all** questions

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

## PHS—301.1

( *Quantum Mechanics-III* )

### GROUP – A

Answer any **two** of the following : 2 × 2

1. Write down the Lippmann-Schwinger equation

( *Turn Over* )

in the position and momentum spaces and identify the Green's functions for both the cases.

2. By acting on the singlet state, check that the permutation operator $P_{12}^{(\text{spin})}$ for a two spin-1/2 particle state can be written as

$$\frac{1}{2}\left(1+\frac{4}{\hbar^2}S_1.S_2\right).$$

3. Write down the Klein-Gordon equation and show that in the non-relativistic limit the equation reduces to the Schrödinger equation.

4. Writing the Dirac matrices as : $\gamma^0 = \beta$ and $\gamma^i = \beta\alpha_i$, show that they satisfy

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} 1_{4\times4}$$

## GROUP – B

Answer any **two** of the following : $4 \times 2$

5. Consider scattering of a particle from a potential $V(r) = \frac{V_0}{\mu}\frac{e^{-\mu r}}{r}$. Compute the differential cross section in the Born approximation (up to the first order).

6. If a system is in the initial state $|i\rangle$ at $t = 0$ and a constant perturbation is switched on at time $t = 0$ so that

$$V(t) = \begin{cases} 0 & (t < 0) \\ V & (t \geq 0) \end{cases}.$$

Find the probability $\left|c_n^{(1)}(t)\right|^2$ of finding the system in the state $|n\rangle$ using first order perturbation theory. Further derive the Fermi's Golden Rule for the transition rate using your result

$$[Use: \lim_{\alpha \to \infty} \frac{\sin^2(\alpha x)}{\pi \alpha x^2} = \delta(x)]$$

7. Write down the Dirac equation in terms of the matrices $(\alpha_i, \beta)$ and obtain the conservation law for the probability $P = \psi^\dagger \psi$ that follows from this equation. Explain how this conservation law is different from the one obtained from the Klein-Gordon equation.

8. Write down the wave functions including the spin of the electrons in an helium atom for the following electronic configuration $(1s)(2s)$ neglecting the $e^2/r_{12}$ interaction term. Next considering the electron-electron interaction, write down the expressions for the difference in the energy levels $\Delta E$, of the electronic states. You do not need to perform any integral. Draw a schematic diagram showing the energy levels of the electronic configurations.

## GROUP – C

Answer any **one** of the following : $8 \times 1$

9. (*i*) For a two state system with unperturbed Hamiltonian $H_0$ and a time-dependent perturbation $V(t)$ given by

$$H_0 = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| \text{ and}$$

$$V(t) = \gamma e^{iwt}|1\rangle\langle 2| + \gamma e^{-iwt}|2\rangle\langle 1|$$

where $\gamma$ is a real constant, set up the time dependent evolution equations in the interaction picture. If the initial state of the system at $t = 0$ is $|1\rangle$, find the probability of obtaining the state $|2\rangle$ after time $t$ by solving the above equations. 6

(*ii*) Given that the scattering amplitude in the partial wave analysis is

$$f(\theta) = \frac{1}{k}\sum_{l=0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_l P_l(\cos\theta),$$

use the optical theorem to find the total cross section $\sigma_{tot}$ in terms of the phase shift $\delta_l$. 2

**10.** (*i*) A particle, initially (i.e., $t \to -\infty$) in its ground state in an infinite potential well whose walls are located at $x = 0$ and $x = a$, is subject at time $t > 0$ to a time-dependent perturbation $V(t) = \epsilon \hat{x} e^{-t^2}$ where $\epsilon$ is a small real number. Calculate the probability (up to the first order in perturbation theory) that the particle will be found in its first excited state after a sufficiently long time (i.e., $t \to \infty$). 4

(*ii*) Compute the expression for the s-wave quantum mechanical scattering cross section of a particle from a hard sphere of radius $R$. The potential corresponding to a hard sphere is given by

$$V(r) = \begin{cases} \infty & (r < R) \\ 0 & (r > R) \end{cases}.$$ 4

## PHS—301.2

### ( *Statistical Mechanics-I* )

### GROUP — A

Answer any **two** of the following : $2 \times 2$

**11.** A spin system has energy $E = \pm\mu_B H$. How negative temperature state is explained ? Plot Entropy vs. energy.

**12.** N free particles of mass $m$ confined to a box (1D) has energy eigen function

$$\psi_k(x) = \sqrt{\frac{2}{L}} \sin kx.$$

Calculate the number of accesible microstates.

**13.** Write down the expression of canonical partition function for 3D Harmonic oscillator.

**14.** Prove that pure state can not evolve into mixed state.

## GROUP – B

Answer any **two** of the following : 4 × 2

**15.** If Hamiltonian $H = {}_i\Sigma A_i p_i^2 + {}_i\Sigma B_i q_i^2 + {}_i\Sigma C_i p_i q_i$ where $q_i$ and $p_i$ are the canonical variables. Prove that $\langle H \rangle = 3NK_BT$ in $3D$.

**16.** Deduce B.E. distribution function from grand-canonical ensemble.

**17.** If energy of classical rotator

$$E = \frac{1}{2I}\left( p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta} \right)$$

show that $C_V = k_B$.

**18.** For a relativistic particle energy $E = C|\vec{p}|$ show that chemical potential

$$\mu(T, P) = k_B T \ln \frac{h^3 c^3 P}{8\pi (k_B T)^4}$$

## GROUP – C

Answer any **one** of the following : 8 × 1

**19.** Calculate the equation of state for N two dimensional quantum Harmonic oscillators.

**20.** Deduce an expression of density matrix for particles in a box in zero potential in position representation. Interpret the results.

**[ Internal Assessment – 10 Marks ]**