# M.Sc. 1st Semester Examination, 2024

## PHYSICS

*( Methods of Mathematical Physics )*

PAPER — PHS-101

*Full Marks* : 25

*Time* : 1 hour

Answer **all** questions

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

## GROUP — A

Answer any **two** questions :     $2 \times 2$

1.  Define Schwarz and Bessel inequality.

2. Evaluate (i) $\varepsilon_{ijk}\, \varepsilon^{ijk}$ (ii) $\delta_{ij}\, \varepsilon_{ijk}$

3. Find the singularities of the function

$$f(z) = \frac{1}{z}\ln(1-z)$$

4. How many independent components does a 2nd rank anti-symmetric tensor have in 3D ?

### GROUP – B

Answer any **two** questions : 4 ×2

5. Evaluate $\displaystyle\int_{-\infty}^{+\infty} \frac{1}{t^2 - R^2} \cos\left(\frac{\pi t}{2R}\right) dt$ (use Residue theorem).

6. Find the Laurent series expansion of

$$f(z) = \frac{1}{(z-1)(z-2)}$$

in the region $1 \le |z| \le 2$ and around $z = 1$.

7. If $A = \begin{pmatrix} 0 & 0 & 0 & \pi/4 \\ 0 & 0 & -\pi/4 & 0 \\ 0 & \pi/4 & 0 & 0 \\ -\pi/4 & 0 & 0 & 0 \end{pmatrix}$

   Find $Tr(e^A)$.

8. Prove that $A^\alpha B_\alpha$ is a scalar.

## GROUP – C

Answer any **one** question :  $8 \times 1$

9. If $\{u_1, u_2, u_3\} = \{1, x, x^2\}$.

   Find three Hermite polynomials using gram-Schmidt orthogonalization in the interval $[-\infty, +\infty]$. Given

   $$\int_{-\infty}^{+\infty} e^{-x^2} H_n(x) H_m(x) dx = 2^n \sqrt{\pi}\, n!\, \delta_{mn}$$

( *Turn Over* )

10. Find the metric tensor and compute various components of the Christoffel symbols if

$$ds^2 = dr^2 \left( 1 - \frac{r^2}{R^2} \right)^{-1} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2).$$

**[ Internal Assessment — 5 Marks ]**