PG/IS/PHS/102/24

M.Sc. 1st Semester Examination, 2024

PHYSICS

(*Classical Mechanics*)

PAPER – PHS-102

Full Marks : 25

Time : 1 hour

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

Answer any **two** of the following questions : 2×2

1. Consider the Lagrangian

$$L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy,$$

where a, b and c are constants. If p_x and p_y are the momenta conjugate to the coordinates x and y respectively, then find the Hamiltonian of the system.

2. Derive Hamilton's canonical equations using Poisson's bracket formalism.

3. Consider the motion of a free particle of mass m. A constant of its motion is

$$F = x - \frac{pt}{m}.$$

Show that $\frac{\partial F}{\partial t} = [H, F]$

4. Find the condition to be satisfied by a, b, c, d so that the transformations $Q = aq + bp$ and $P = cq + dp$ is canonical.

GROUP – B

Answer any **two** of the following questions : 4×2

5. Derive the equation of motion for a simple pendulum using the variational principle.

6. The Hamiltonian of a one dimensional simple harmonic oscillator is

$$H = \frac{p^2}{2m} + \frac{kq^2}{2}.$$

As a result of the canonical transformation generated by $F = Cq^2 cotQ$, where $C = \sqrt{mk}/2$ is a constant, find the Hamiltonian in the new coordinate Q and momenta P.

7. Show that the transformation $Q = (q^2 + p^2)/2$ and $P = -\tan^{-1}\left(\frac{q}{p}\right)$ is canonical.

8. The Lagrangian of motion of a two dimensional isotropic harmonic oscillator is given by

$$L = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{K}{2}\left(x^2 + y^2\right).$$

Given $F = \dot{x}\dot{y} + a\alpha y$ is a constant of motion. Find α.

GROUP – C

Answer any **one** of the following questions : 8×1

9. (*a*) The Lagrangian L for a charged particle with charge e moving in an electromagnetic field is given by

$$L = \frac{1}{2} m\dot{\vec{r}}^2 - e\varphi(r,t) + e\vec{A}(r,t).\dot{\vec{r}},$$

find the Hamiltonian.
where :
m is the mass of the particle,
$\dot{\vec{r}}$ is the velocity of the particle,
$\varphi(r, t)$ is the scalar potential,
$\vec{A}(r,t)$ is the vector potential.
e is the charge of the particle 4

(*b*) The Lagrangian of a system moving in three dimension is

$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - \frac{1}{2}Kx^2 - \frac{1}{2}K(y+z)^2.$$

Assuming $L_x = yp_z - zp_y$, determine $\frac{dL_x}{dt}$ 4

10. A particle is moving in one dimension with a potential qa^2 where a is constant. Write down the Lagrangian for the system. Solve the equation of motion using Hamilton Jacobi method. What is the physical significance of Hamilton's principle function ? What is the physical significance of Hamilton's principle function and Hamilton's characteristics function ?

$$1+4+\left(1\frac{1}{2}+1\frac{1}{2}\right)$$

[Internal Assessment — 5 Marks]
