M.Sc. 1st Semester Examination, 2024

PHYSICS

(*Quantum Mechanics-I*)

PAPER – PHS-103

Full Marks : 25

Time : 1 hour

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

GROUP – A

Answer any **two** of the following : 2×2

1. If $\hat{p}$ is the momentum operator and given that $\langle x \mid p \rangle = (2\pi\hbar)^{-1/2} e^{ipx/\hbar}$, rewrite the expression $\langle x \mid \hat{P} \mid \psi \rangle$ in terms of $\psi(x)$.

2. Consider a complete and orthonormal basis $\{|1\rangle, |2\rangle\}$. The action of an operator $\hat{L}$ on the basis kets is given by

$$\hat{L}|1\rangle = 2i\hbar|2\rangle\ ;\ \hat{L}|2\rangle = -2i\hbar|1\rangle - 3\hbar|2\rangle.$$

What possible value are obtained when $\hat{L}$ is measured ?

3. Obtain the matrix representation of the time evolution operator $\hat{U}(t)$ corresponding to the Hamiltonian $H = \in \sigma_x$. Here $\in$ is a constant having dimensions of energy and σ_x is a Pauli matrix.

4. For a 1D harmonic oscillator, show that $\frac{(a^\dagger)^2}{\sqrt{2!}}|0\rangle$ is an eigenstate of the number

operator $\hat{N} = a^\dagger a$, where $a^\dagger$ and a are creation and annihilation operators respectively. What is the corresponding eigenvalue ?

GROUP – B

Answer any **two** of the following : 4×2

5. For a one dimensional harmonic oscillator obtain the expression

$$\left\langle \left(\Delta\hat{X}\right)^2 \right\rangle \left\langle \left(\Delta\hat{P}\right)^2 \right\rangle$$

for the state $\psi_n(x) = \langle x \mid n \rangle$.

6. Suppose a system is in the state $|+, z\rangle$ which is an eigenstate of $\hat{S}_z$ with eigenvalue $\hbar/2$ and

$$\hat{S}_y = i\hbar/2(-|+,z\rangle\langle -,z| + |-,z\rangle\langle +,z|).$$

(*i*) Find the probabilities of finding the system in the eigenstates $|\pm, y\rangle$ of $\hat{S}_y$.

(*ii*) Find the unitary matrix U such that

$$U|+,z\rangle = |+,y\rangle.$$

7. For a one dimensional harmonic oscillator, using the fact that $\langle x | a | 0 \rangle = 0$, where a is the annihilation operator, obtain the normalized ground state wave function $\psi_0(x)$. Explain how you would obtain the wave function for the first excited state $\psi_1(x)$ from $\psi_0(x)$.

8. Consider two operators $\hat{A}$ and $\hat{B}$ whose matrix representations are given as

$$[\hat{A}] = \begin{pmatrix} a & 0 & 0 \\ 0 & -a & 0 \\ 0 & 0 & -a \end{pmatrix} \qquad [\hat{B}] = \begin{pmatrix} b & 0 & 0 \\ 0 & 0 & -ib \\ 0 & ib & 0 \end{pmatrix}$$

Show that $\left[\hat{A}, \hat{B}\right] = 0$ and find the simultaneous eigenkets of both $\hat{A}$ and $\hat{B}$.

GROUP – C

Answer any **one** of the following : 8×1

9. (*i*) The Hamiltonian operator for a spin-1/2 system is given by $\hat{H} = \omega \hat{S}_z$ where ω is a real constant. The state of the system at $t = 0$ is $|+, x\rangle$ find the expectation values of the operators $\hat{S}_x, \hat{S}_y, \hat{S}_z$ as a function of time using the Schrödinger picture. 6

(*ii*) Suppose $\hat{T}(\Delta x)$ is a space translation operator in one dimension. By acting on the state $|x\rangle$, find the commutator,

$$\left[\hat{X}, \hat{T}(\Delta x)\right].$$ 2

10. (*i*) Consider the following state :

$$|\psi(0)\rangle = A(3|0\rangle + 4|1\rangle)$$

where $|n\rangle$ is the normalized n-th excited state of a one dimensional harmonic oscillator with energy eigenvalue

$$E_n = \hbar\omega(n + 1/2).$$

Find the constant A so that $|\psi(0)\rangle$ is normalized. Suppose the system is evolved in time using the Hamiltonian for a one dimensional harmonic oscillator. Find the expectation value $\langle \hat{X} \rangle(t)$ using the Schrödinger picture. 4

(*ii*) Consider a free particle of mass m in one dimension. Solve the Heisenberg equations of motion and evaluate the commutator $\left[\hat{X}(t), \hat{X}(0)\right]$. 4

[Internal Assessment — 5 Marks]
