 **PG/IVS/MTM/498(C)/25(Pr.)**

# M.Sc. 4th Semester Examination, 2025

## APPLIED MATHEMATICS

( *Computational and Semi-Analytical Method : Skill Development Course* )

[ Practical ]

PAPER – MTM-498C

*Full Marks* : 25

*Time* : 2 hours

*The figures in the right hand margin indicate marks*

*Symbols/Notations have their usual meaning.*

**Questions are distributed by lottery.**

GROUP – A

Answer **one** question : 6 × 1

1. Consider the following one-dimensional heat-conduction equation :

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

An insulated rod of length 2m initially has a temperature of $T(x, 0) = 30^{\circ}$C, and at $t = 0$ hot reservoirs ($T = 70^{\circ}$C) are brought into contact with the two ends, A and B. Thermal conductivity of rod ($\alpha$) is 16 W/m.K. Write a MATLAB code using Forward-Time-Central-Space (FTCS) scheme to find the numerical solution of subsequent temperature $T(x, t)$ of any point in the rod with time-step 0.0001. Take the appropriate space-step so that the solution is stable. Show the temperature $T(x, t)$ graphically at different times $t = 1, 3, 5$. Also the show $T(x, t)$ graphically at times $t = 1, 3, 5$ when $\alpha$ is 30 W/m.K.

2. For the above problem, write a C-program/ MATLAB code using Crank-Nicolson Method to find the numerical solution of subsequent temperature $T(x, t)$ of any point in the rod with time-step 0.001. Take the appropriate space-step so that the solution converges.

3. Consider the problem of source-free heat conduction in an insulated rod whose ends are maintained at constant temperatures of $20^\circ$C and $50^\circ$C respectively. The *one* dimensional problem is governed by

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0$$

Write a C-program/MATLAB code to calculate the steady state temperature distribution in the rod. Thermal conductivity $k$ equals 700 W/m.K, Cross-sectional area A is $10 \times 10^{-3}$ m$^2$. Use Finite Volume Method (FVM) and explicit scheme.

## GROUP—B

Answer **one** question : 9 × 1

4. Consider the following one-dimensional Burger's equation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\frac{\partial^2 u}{\partial x^2}$$

with the initial condition $u(x,0)=2x, t>0$. Write a Mathematica Code to find the numerical solution of the above problem using the variational Iteration Method (VIM) with linear operator $L(u)=\frac{\partial u}{\partial t}$. Show the solution graphically.

5. Consider the following hyperbolic nonlinear problem

$$\frac{\partial u}{\partial t}=u\frac{\partial u}{\partial x}, \quad 0\le x\le 1, \quad 0\le t\le 1,$$
$$u(x,0)=\frac{x}{10}$$

Write a Mathematica Code to find the 5th order approximate solution of the above problem using the Adomian Decomposition Method (ADM) with linear operator $L(u) = \frac{\partial u}{\partial t}$.

6. Consider the following nonlinear problem

$$f''(x) + f'^2(x) = 0$$
$$f(0) = 1 f'(0) = 2$$

Write a Mathematica Code to find the 5th order approximate solution of the above problem using the Homotopy Perturbation Method (HPM) with linear operator $L(f) = f''$. Exact solution of this problem is $f(x) = 1 + \ln(2x + 1)$. Show the comparison between the numerical solution and exact solutions graphically.

7. Consider the following nonlinear problem

$$y'' + y(y') = 0, y(0) = 1, y'(0) = -1$$

Write a Mathematica Code to find the 5th order approximate solution of the above problem using the Homotopy Analysis Method (HPM) with linear operators $L(f) = y'' + y'$ and $L(f) = y''$. Show the comparison between these two solutiuons graphically.

8. Consider the second order non-linear differential equation with and exponential nonlinearity as $y + y^2 = 0$ $y(0) = 1$ and $y'(0) = 0$. Using the Adomain Decomposition Method, find the 2nd order approximated solution.

9. Viva and Note book : 10 Marks