M.Sc. 2nd Semester Examination, 2025

APPLIED MATHEMATICS

(*Fluid Mechanics*)

PAPER – MTM-201

Full Marks : 50

Time : 2 hours

Answer all questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Symbols/Notations have their usual meaning

1. Answer any *four* questions : 2×4

   (*a*) Write three areas of study of fluid mechanics and show them graphically.

(*Turn Over*)

What is steady and unsteady flows ? Discuss them mathematically and graphically.

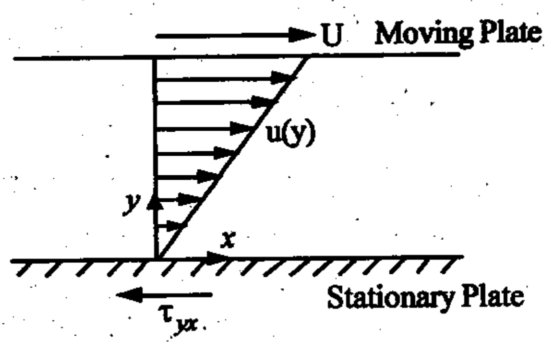
(*b*) For a Newtonian incompressible fluid flow (of viscosity $\gamma \sim 10^{-6}\,\text{m}^2/\text{s}$) over a circular cylinder of radius 2cm with inlet velocity $U = 1\,m/s$, calculate the Reynolds number.

(*c*) Find the substantial derivative of the steady state velocity field represented by the velocity vector $\vec{V} = (x^2, y, -z^2)$.

(*d*) Consider a Newtonian incompressible fluid flow within a Cavity as shown below :

Four walls of the cavity are impermeable. Left, right and bottom walls are fixed while top wall is moving with velocity U. Write the boundary conditions for tangent and normal component of the velocity.

(*e*) Following the velocity profile for Couettee flow between two plates.



Draw the velocity profile for Couette-Poiseuille flow for different nondimensional pressure gradient $P = \frac{h^2}{2\mu U}\left(-\frac{dp}{dx}\right)$ where μ is the viscosity, h is the distance between two plates and p is the pressure.

(*f*) Derive the hydrostatic equation and hence estimate the pressure experienced by a fish at a depth of 50 m.

2. Answer any *four* questions : 4×4

(*a*) Write all the possible boundary conditions for tangential and normal components of velocity, and temperature.

(*b*) Write the set of governing equations for the boudnary layer flow along a flat plate along with the proper boundary conditions for the above set of equations. Show that the x-component of the momentum equation applied at the edge of the boundary layer reduced to the Bernoulli equation. Finally write the governing equations for outside the Boundary layer.

(*c*) Make the energy equation

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)$$

into non-dimensional form (in terms of Reynolds number $Re=\frac{UL}{\nu}$ and *Prandtl* $Pr=\frac{C_p\mu}{k}$ where μ is the viscosity and C_p is the specific heat) with the help of characteristics length, velocity and temperature L, U and T_0, respectively, and symbols have their usual meaning.

(*d*) Examine whether the motion specified by $q=\frac{\alpha^2(-y\hat{i}+x\hat{j})}{x^2+y^2}$, ($\alpha$ = constant) is a possible motion for an incompressible fluid. If so, determine the equations of streamlines.

(*e*) For the Couette flow in a channel :

(*i*) draw the flow configuration

(*ii*) write the necessary assumptions

(*iii*) deduce the approximate equation from the complete set of Navier-Stokes equation for 2D laminar incompressible viscous flow

(*iv*) write the required boundary condition on the flow configuration. Finally, solve the modified equation and show the velocity profile graphically.

(*f*) Derive the equation of continuity, in Cartesians, for the flow of a compressible fluid.

3. Answer any *two* questions : 8×2

(*a*) (*i*) Consider the x-component of Navier-Stokes equations in conservation form as

$$\frac{\partial(\rho u)}{\partial t}+\nabla\cdot(\rho u V)=-\frac{\partial p}{\partial x}+\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}+\rho f_x,$$

where the symbols have their usual meaning. For Newtonian fluids, write the expression for τ_{xx}, τ_{yx} and τ_{zx}. Then using the Stokes hypothesis, simplify the above for compressible as well as incompressible flow.

(*ii*) Discuss briefly the similarity/dissimilarity between the x-momentum and simplified form of energy equations for the steady-state, two dimensional flow of an incompressible fluid with constant properties. (6 + 2)

(*b*) (*i*) Stating necessary the assumptions of boundary layer theory, derive the set of governing equations for the boundary layer flow along a flat plate. Also write the proper boundary conditions for the above set of equations.

(*ii*) Find the thickness of the boundary layer for an incompressible viscous flow over a flat plate of length 10 meter at which the Reynold number is 10^4. 6 + 2

(*c*) (*i*) Discuss the role of the non-linear terms of Reynolds average Navier-Stokes (RANS) equation.

(*ii*) Define Rossby Number, and discuss the cases of its low and high values. 6 + 2

(*d*) (*i*) Write all the four forms of the continuity equation : Integral-Conservation, Integral-Nonconservation, Differential-Conservation and Differential-Noconservation. Finally convert the Differential-NonConservation form to that of Differential-Conservation.

(*ii*) For typical horizontal length scale (L) of 700km, horizontal speeds (U) are of the order of $0.1 ms^{-1}$ and a vertical scale length (H) of 1100m, estimate a typical vertical speed(W). 4 + 4

[Internal Assessment — 10 Marks]