M.Sc. 2nd Semester Examination, 2025

APPLIED MATHEMATICS

(*Numerical Analysis*)

PAPER—MTM-202

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

1. Answer any *four* questions of the following : 2×4

(*a*) Is the following function a cubic spline ? Justify.

$$y(x)=\begin{cases} x^3-4x^2+5x-2, & 1\le x\le 3 \\ x^3+x^2+25x+43, & 3\le x\le 4 \end{cases}$$

(Turn Over)

(*b*) Express the polynomial $x^4 + 7x^2 - 5x$ in terms of Chebyshev polynomials.

(*c*) What are the limitations of power method for eigenvalue problem ?

(*d*) Calculate the total number of arithmetic computations required to solve a system of linear equations using the LU-decomposition method.

(*e*) Discretise the following equation

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$$

using the finite difference method.

(*f*) Define absolute stable and relatively stable of an ODE.

2. Answer any *four* questions of the following : 4 × 4

(*a*) Approximation $f(x) = e^{2x}$ upto 3rd order

Chebyshev approximate over the interval [0, 1].

(*b*) Find the value of

$$\int_0^2 \frac{x}{1+x^2}dx$$

using 4-point Gauss-Legendre quadrature formula.

(*c*) Analyse the stability of 2nd order Runge-Kutta method.

(*d*) Explain a finite difference method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}, \quad t>0, \quad 0<x<1$$

where initial conditions $u(x, 0)=f(x)$

and $\left(\frac{\partial u}{\partial t}\right)_{(x,0)}=g(x), 0<x<1$ and

boundary conditions $u(0,t)=\varphi(t)$

and $u(1,t)=\psi(t), t\geq 0.$

(*e*) Describe the Newton-Raphson method for the following system of equations :

$$f(x,y)=0,\ g(x,y)=0.$$

(*f*) Solve the following system of equations $x=(8x-4x^2+y^2+1)/8$ and $y=(2x-x^2+4y-y^2+3)/4$ starting with $(x_0, y_0)=(1.1, 2.0)$, using Seidel iteration method.

3. Answer any *two* questions of the following : 8 × 2

(*a*) Define spline interpolation. Fit a cubic spline that passes through

$$(-1,0),\left(0,\frac{1}{2}\right),(1,2),\left(2,\frac{3}{2}\right)$$

with the conditions $y''(-1)=y''(2)=0$. 2 + 6

(*b*) Derive the Milne's predictor and corrector formulae. Explain how many staring values are required to obtain the solution by this method and how the starting points are obtained. 6 + 2 (*c*) Describe Braistow's method to find all roots of an algebraic equation of degree n.

(*d*) Explain the successive overrelaxation method to solve a system of linear equations.

[Internal Assessment — 10 Marks]