 PG/IIS/MTM/205/25

# M. Sc. 2nd Semester Examination, 2025

## APPLIED MATHEMATICS

(*General Theory of Continuum Mechanics*)

PAPER – MTM-205

*Full Marks* : 50

*Time* : 2 hours

Answer **all** questions

*The figures in the right hand margin indicate marks*

*Candidates are required to give their answers in their own words as far as practicable*

## GROUP – A

1. Answer any four questions : 2 × 4

(*i*) Find the relation between α and β such that the small deformation defined by $u_1 = \alpha x_1 + 3x_2, u_2 = x_1 - \beta x_2$ and $u_3 = 3x_3$ is isochoric. 2

(*ii*) Define principal strain and principal direction of strain. 2

(*iii*) The components of a stress dyadic at a point referred to the $(x_1, x_2, x_3)$ system are given by

$$(T_{ij}) = \begin{pmatrix} 12 & 9 & 0 \\ 9 & -12 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

For this state of stress, determine the maximum shear stress. 2

(*iv*) What is the concept of stress quadric ? 2

(*v*) Show that the difference between two values of a stream function at two points represents the flux of a fluid across any curve joining these two points. 2

(*vi*) Establish the relation among bulk modulus, Young's modulus and Poisson's ratio. 2

## GROUP—B

2. Answer any *four* questions : 4 × 4

(*i*) Derive the extensional strain tensor. The strain tensor at a point is given by

$$\left(E_{ij}\right)=\begin{pmatrix}3 & 2 & 1\\ 2 & 0 & -1\\ 1 & -1 & 2\end{pmatrix}.$$

Determine the extension of the line element in the direction of $\left(\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$. 2 + 2

(*ii*) What is significance of an image ? Find the image of a source with respect to a straight line. 4

(*iii*) For the following stress distribution

$$\left(T_{ij}\right)=\begin{pmatrix}x_1+x_2 & T_{12} & 0\\ T_{12} & x_1-2x_2 & 0\\ 0 & 0 & x_2\end{pmatrix},$$

find $T_{12}(x_1, x_2)$ in order that stress distribution is in equilibrium with zero body force and the stress vector on $x_1 = 1$ is given by $\vec{T}^n = (1 + x_2)\vec{e}_1 + (2 - x_2)\vec{e}_2$. 4

(*iv*) Show that the following is the possible form of the boundary surface of a liquid motion :

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1.$$ 4

(*v*) Derive the compatibility equation for strain deformation. 4

(*vii*) What is the concept of elasticity ? Show that the principal directions of strain at each point in a linearly elastic isotropic body must be coincident with the principal directions of stress. 4

GROUP—C

3. Answer any *two* questions : 8 × 2

(*i*) What is the concept of stress vector ? Derive the extremum normal stress at a point of the continuum. 2 + 6

(*ii*) (*a*) Find the integral of the Euler equation of motion for perfect fluid stating necessary assumptions when the flow is rotational and steady. 6

(*b*) If $\phi = (x_1 - t)(x_2 - t)$ be the velocity potential of a two-dimensional irrotational motion of a continuum, then show that stream lines at time $t$ are

$$(x_1 - t)^2 - (x_2 - t)^2 = \text{constant.}$$ 2

(*iii*) State and prove energy equation of perfect fluid. Give examples of irrotational and rotational flows. 7 + 1 (*iv*) Define isotropic linear elastic body. Derive equation of motion for an isotropic linear elastic body. Under what conditions this body shows wave equations for displacement and then derive it. 1 + 4 + 3

**[ Internal Assessment — 10 Marks]**

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