M. Sc. 2nd Semester Examination, 2025

APPLIED MATHEMATICS

(*General Topology*)

PAPER – MTM-206

Full Marks : 25

Time : 1 hour

Answer all questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

GROUP—A

Answer any **two** questions : 2 × 2

1. Show that a closed subspace of a normal space is normal.

2. Show that the order topology on $\mathbb{Z}_+$ is the discrete topology.

(*Turn Over*)

3. Define completely regular space with example.

4. Is the collection $\tau = \{U : X - U$ is infinite or empty or all of $X\}$ a topology on X?

GROUP—B

Answer any **two** questions : 4×2

5. Let A be a subset of a topological space X. Then show that $x \in \bar{A}$ if and only if every open set U containing x intersects A. 4

6. Let A be a proper subset of X and B be a proper subset of Y. If X and Y are connected show that $(X \times Y) - (A \times B)$ is connected. 4

7. Define convergence of a sequence of points in a topological space X. Show that in a Hausdorff space X, a sequence can converge to at most one point of X. 1 + 3

8. Let X be a Hausdorff topological space and F be a compact subspace of X. Show that F is closed in X. 4

GROUP—C

Answer any **one** question : 8 × 1

9. (*i*) Let $f: X \to Y$ be a function. Then show that the followings are equivalent : 4

(*a*) for every closed set V of Y, $f^{-1}(V)$ is closed in X,

(*b*) for every $A \subseteq X, f\left(\overline{A}\right) \subseteq \overline{f(A)}$.

(*ii*) Let $\{A_\alpha\}$ be a collection of connected subspaces of X and A be a connected subspace of X. Show that if $A \cap A_\alpha \neq \phi$ for all α, then $A \cup \left(\cup_\alpha A_\alpha\right)$ is connected. 4

10. (*i*) Show that compactness implies limit point compactness. 3

(*ii*) Show that $\mathbb{R}^{\omega}$ in the product topology is connected. 5

[Internal Assessment — 05 Marks]
