M. Sc. 4th Semester Examination, 2025

APPLIED MATHEMATICS

PAPER – MTM-404 (A & B)

Full Marks : 50

Time : 2 hours

Answer **all** questions

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PAPER — MTM-404(A)

(Elective Paper : Non-Linear Optimization)

[*Marks* : 25]

1. Answer any *two* questions : 2×2

    (*a*) Define degree of difficulty of all types of a geometric programming problem.

(b) Define differentiable convex and concave functions.

(c) Define Nash equilibrium strategy and Nash equilibrium outcome in pure strategy of a bi-matrix game.

(d) State Wolfe's duality theorem in connection with duality in quadratic programming.

2. Answer any *two* questions : 4×2

(a) Define the following :

(i) Minimization problem;

(ii) Local minimization problem;

(iii) Kuhn-Tucker saddle point problem;

(iv) Fritz-John saddle point problem.

(b) State and prove Weak duality theorem in connection with duality in non-linear programming.

(c) Minimize the following using geometric programming

$$f(x) = 16x_1 x_2 x_3 + 4x_1 x_2^{-1} + 2x_2 x_3^{-2} + 8x_1^{-3} x_2,$$

$$x_1, x_2, x_3 > 0.$$

(d) Define multi-objective non-linear programming problem. Define the following in terms of multi-objective non-linear programming problem :

(i) Complete optimal solution

(ii) Pareto optimal solution

(iii) Local Pareto optimal solution 8×1

(iv) Weak Pareto optimal solution.

3. Answer any *one* question :

(a) (i) Use the chance constrained programming to find an equivalent deterministic problem to following stochastic programming problem, when c_j is a random variable :

Minimize $F(x) = \sum_{j=1}^{n} c_j x_j$

subject to $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$

$x_j \geq 0, i, j = 1, 2, \cdots, n.$

(ii) What is the necessity of constraint qualifications ? Define two constraint qualifications. 4 + 4

(b) (i) Prove that a pair $\{y^*, z^*\}$ constitutes a mixed strategy Nash equilibrium solution to a bimatrix game (A, B) if and only if, there exists a pair $\{p^*, q^*\}$ such that $\{y^*, z^*, p^*, q^*\}$ is a solution of the following bilinear programming problem :

Minimize $[y'Az + y'Bz + p + q]$

subject to $Az \geq -pl_m,$

$B'y \geq -ql_n,$

$y \geq 0, z \geq 0, y'l_m = 1, z'l_n = 1.$

(ii) State and prove Kuhn-Tucker station-
ary point sufficient optimality theorem.

5 + 3

[Internal Assessment — 05 Marks]

PAPER — MTM-404(B)

(*Elective Paper : Dynamical Meteorology-II*)

[*Marks* : 25]

1. Answer any *two* questions :   2 × 2

(a) What is the concept of geodynamical
paradox in the atmosphere ?

(b) If the height of the 50kPa pressure surface
decreases by 10m northward across a
distance of 500 km, what is the pressure
- gradient force ?

(c) Find geostrophic wind components at a
location where $\rho = 1.2 \, kg \, . \, m^{-3}$ and
$f_c = 1.1 \times 10^{-4} \, s^{-1}$ where pressure decreases
by 2kPa for each 800km of distance north.

(d) What is the concept of cyclostrophic wind in the atmosphere ?

2. Answer any *two* questions : 4 × 2

(a) Derive the tangential velocity of a tropical cyclone when it reaches at a distance R from eye - centre of the cyclone. Suppose that air at a latitude of $30°$ initially has no rotation as it is drawn toward a low - pressure center 500 km away. When the air reaches 200 km from the low center, what will be its tangential velocity ? Given that the angular velocity of the earth is $0.729 \times 10^{-4} s^{-1}$. 2 + 2

(b) Write down the vertical momentum equation in cartesian form for an air parcel in the atmosphere. Hence derive the hydrostatic equilibrium in the atmosphere considering necessary assumptions. 1 + 2 + 1

(c) Explain the physical significance of thermal wind in the atmosphere. While driving

north a distance of 500 km, your car thermometer shows the outside air temperature decreasing from $20°C$ to $10°C$. How does the theoretical geostrophic wind changes with altitude ? 2 + 2

(d) Derive the horizontal momentum equation of an air parcel in the atmosphere in natural co-ordinate system.

3. Answer any *one* question : 8 × 1

(a) Derive the vorticity equation of an air parcel in the atmosphere.

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