M. Sc. 4th Semester Examination, 2025

APPLIED MATHEMATICS

PAPER – MTM-405 (A, B & C)

Full Marks : 25

Time : 1 hour

The figures in the right hand margin indicate marks

Candidates are requested to give their answers in their own words as far as practicable

Elective Paper – MTM-405A

(*Operational Research Modelling-II*)

[*Marks* : 25]

1. Answer any *two* questions : 2×2

   (*a*) What do you mean by prefix-free encoding ? What is the importance of this encoding ?

(Turn Over)

(*b*) Define reliability. How does it differ from probability ?

(*c*) Prove that entropy function is continuous.

(*d*) What are the differences between the reliability of a device and a system of components ?

2. Answer any *two* questions : 4×2

(*a*) For any two independent messages X, Y, prove that $H(X,Y) = H(X) + H(Y)$.

(*b*) Suppose a system contains a primary element and a standby element. Let λ_p and λ_d represent the failure rates of the primary and stand-by elements. Find the reliability of this system. Also, find the system reliability and MTBF when $\lambda_p = \lambda_d = \lambda$.

(*c*) In a certain community 25% of all girls are blondes and 75% of all blondes have blue eyes. Also, 50% of all girls in the community have blue eyes. If you know that a girl has blue eyes, how much additional information do you get by being informed that she is blonde ?

(*d*) A system is described by $\dot{x}_1 = -2x_1 + u$ and the control $u(t)$ will be chosen to minimise $\int_0^1 u^2\, dt$. Show that the optimal control which transfers the system from $x_1(0) = 1$ to $x_1(1) = 1$ is given by $u^* = -\dfrac{4e^{2t}}{e^4 - 1}$.

3. Answer any *one* question : 8 × 1

(*a*) (*i*) In a system, there are *n* number of components connected in parallel with reliability $R_i(t)$; $i = 1, 2, ..., n$. Find the reliability of the system. If

$R_1(t) = R_2(t) = \cdots = R_n(t) = e^{\lambda t}$, then what will be the expression of system reliability ?

(*ii*) Define system Reliability. Find the reliability of a system with two components of which one is a stand-by. The components are connected in parallel.

(*b*) Define marginal and conditional entropies. A transmitter has a character consisting of five letters $(x_1, x_2, x_3, x_4, x_5)$ and the receiver has a character consisting of four letters (y_1, y_2, y_3, y_4). The joint probability for the communication is given below :

$P(x_i, y_j)$	y_1	y_2	y_3	y_4
x_1	0.25	0	0	0
x_2	0.10	0.30	0	0
x_3	0	0.05	0.10	0
x_4	0	0	0.05	0.10
x_5	0	0	0.05	0

Determine $H(X)$, $H(Y)$, $H(X,Y)$ and $H(Y=X)$.

[Internal Assessment — 05 Marks]

Elective Paper — MTM-405B

(*Dynamical Oceanology : Coastal Processes*)

[*Marks* : 25]

1. Answer any *two* questions : 2×2

(*a*) State the basic physical laws used in oceanography.

(*b*) Write down the equations of conservation of momentum and energy for tsunamis.

(*c*) Explain the wave breaking types 'spilling' and 'surging' with graphical representation.

(*d*) Prove that the bed shear stress is harmonic in time and lags the free surface displacement by 45 degrees for the laminar boundary layer in a viscous fluid.

2. Answer any *two* questions : 4 × 2

(*a*) What role does wave steepness play in wave breaking ? How does wave breaking affect coastal erosion ?

(*b*) Describe all types of motions and forces in oceanography.

(*c*) Discuss the properties of tsunamis. Write also the empirical formula for run-up over a beach or a structure slope from Kaplan laboratory experiments.

(*d*) Determine the wave lengths of the two possible modes of wave propagation over an infinite deep mud layer, with

$\frac{\rho_2}{\rho_1} = 1.2$; while, the overlying water column is 4.6m in depth and wave period is 8 s.

3. Answer any *one* question : 8 × 1

(*a*) Prove that the dispersion relation of water waves over a viscous mud region is

$$\left(\sigma^2 - gk\right)\left[\sigma^2\left(\frac{\rho_2}{\rho_1} + \tanh kh\right) - \left(\frac{\rho_2}{\rho_1} - 1\right)gk \tanh kh\right] = 0,$$

symbols have their own meaning. Hence, find out two possible roots exist for waves propagating in the positive longitudinal direction. Also, plot the dispersion relationship for waves over an infinite deep water. 5 + 2 + 1

(*b*) (*i*) Explain, the terms 'wave diffraction' and 'wave shoaling'.

(*ii*) Sea waves are striking approximately normally against a semi-infinite breakwater in 6m deep water with wave height 2.5m and time period 10sec. What are the values of wave height and time period at a location 400 m behind and 400 m on the lee of the breakwater ? 2 + 2 + 4

[Internal Assessment — 05 Marks]

Elective Paper — MTM-405C

(*Computational and Semi-Analytical Methods*)

[*Marks* : 25]

1. Answer any *two* questions : 2 × 2

(*a*) Discuss the boundary condition for pressure correction at the left boundary of the computational domain when velocity components are specified at the boundary.

(*b*) Write the homotopy equations for Homotopy Perturbation Method (HPM) and Optimal Homotopy Analysis Method (OHAM) and then write one major difference between these two equations.

(*c*) Let the nonlinear part is of the form $N(u) = u^2$. Calculate first two Adomain polynomials.

(*d*) In order to solve the following two non-linear equations, write the linear auxiliary operator in terms of their auxiliary roots under Optimal and Modified Homotopy Perturbation Method (OM-HPM)

$$y'' + y(y') = 0, y(0) = 1, y'(0) = -1 \text{ and}$$

$$y''' - yy'' + (y')^2 = 0, y(0) = 1, y'(0) = 0, y''(0) = 2$$

2. Answer any *two* questions : 4 × 2

(*a*) Write down the complete flowchart for Semi Implicit Method for Pressure Linked Equations Revised (SIMPLER).

(*b*) Solve the nonlinear second-order ODE : $y'' + y^2 = 0$, with initial conditions $y(0) = 1$ and $y'(0) = 0$ using the Adomain Decomposition Method (ADM) up to 2nd order approximation.

(*c*) (*i*) Write down the development of Homotopy Analysis Method (HAM).

(*ii*) What are the four pillars of convergence of solution of HAM ? 2 + 2

(*d*) Consider the following two-dimensional Burger's equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right),$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{R}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right),$$

with the initial conditions :

$$u(x,y,0) = x + y,$$
$$v(x,y,0) = x - y.$$

With the linear operator $L(u) = \frac{\partial u}{\partial t}$ and $L(v) = \frac{\partial v}{\partial t}$, write the Variation Iteration scheme for the above problem.

3. Answer any *one* question : 8 × 1

(*a*) (*i*) Let the discretised equation for x- and y-momentum equations are, respectively,

$$a_j^u u_{j-1,k}^{n+1} + b_j^u u_{j,k}^{n+1} + c_j^u u_{j+1,k}^{n+1} = d_j^u - \Delta y\left(p_{j+1,k}^{n+1} - p_{j,k}^{n+1}\right) \text{ and}$$
$$a_j^v v_{j-1,k}^{n+1} + b_j^v v_{j,k}^{n+1} + c_j^v u_{j+1,k}^{n+1} = d_j^v - \Delta x\left(p_{j+1,k}^{n+1} - p_{j,k}^{n+1}\right).$$

Using these, derive the Pressure Poisson equation. Symbols have their usual meaning.

(*ii*) Find out the Lagrange's multiplier for the problem $u' = 2u - u^2 + 1$ for the purpose of Variation Iteration scheme. 6 + 2 (*b*) (*i*) Let us consider a non-homogeneous advection equation as $u_t + uu_x = x$, subject to the initial condition $u(x,0) = 2$. Find the first order Homotopy Pertubation Method (HPM) solution for the above equation with the linear operator $L(u) = u_t$.

(*ii*) Write down the expression for the exact square residual and average square residual for k-th order Homotopy approximation $u_k(x)$. 6 + 2

[Internal Assessment — 05 Marks]