 PG/IVS/MTM/498(A)/25(Pr.)

M.Sc. 4th Semester Examination, 2025

APPLIED MATHEMATICS

(*LAB : OR Methods using MATLAB and LINGO*)

[Practical]

PAPER – MTM-498A

Full Marks : 25

Time : 2 hours

The figures in the right hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

Solve one problem using MATLAB and another problem with LINGO

The questions are to be distributed by using a Lottery

(Marks distribution : MATLAB 9; LINGO 6; Viva & Lab Note Book : 5; Field Visit : 05)

(*Turn Over*)

1. Let

   p = shortage cost per unit short per unit time short,

   S = inventory level just after a batch of Q units is added to inventory,

   Q-S = shortage in inventory just before a batch of Q units is added.

   Production or ordering cost per cycle (OC) = K + cQ.

   Holding cost per cycle (HC) = $\frac{hS^2}{2a}$.

   Shortage cost per cycle (SC) = $\frac{p(Q-S)^2}{2a}$.

   Total cost per cycle $TC = OC + HC + SC$

   Total cost per unit time $ATC = \frac{TC}{t}$, $t = \frac{Q}{a}$.

   Find the optimum values of Q, S, t, when $K = 12000$, $h = 0.30$, $a = 8000$, $p = 1.10$, such that the total cost per unit time is minimum.

2. A contractor has to supply 120 bearings per day to an automobile manufacturer. The manufacturer can produce 260 bearings per day in a production run. The cost of holding a bearing in stock for one year is Rs. 2, and the set-up cost of a production run is Rs. 180. Write a program in MATLAB to find the following :

   (*i*) Find the lot size (Q) that minimize the cost.

   (*ii*) Determine the total optimal cost

   (*iii*) Cycle time (t)

3. Solve the following non-linear programming problem by dynamic programming method using MATLAB :

   Min $z = y_1^2 - y_1 y_2 + y_1 y_3 + y_4^2$

   Subject to $y_1 + 2y_2 + 3y_3 + 4y_4 \geq 80;$

   $-y_1 + y_2 + 4y_3 + 2y_4 \leq 90;$

   $y_1, y_2, y_3, y_4, >= 0;$

4. Assume the following notations and formula :

λ : Arrival rate

μ : Service rate

P_0 : Probability of no customer in the system $= \dfrac{\mu - \lambda}{\mu}$

L_s : Expected (average) number of units in the system $= \dfrac{\lambda}{\mu - \lambda}$

L_q : Expected (average) queue length $= \dfrac{\lambda^2}{\mu(\mu - \lambda)}$

W_q : Mean (or expected) waiting time in the queue (excluding service time) $= \dfrac{\lambda}{\mu(\mu - \lambda)}$

W_S : Expected waiting time in the system (including service time) $= \dfrac{1}{\mu - \lambda}$

In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time distribution is also exponential with an average of 36 minutes. Write a code in MATLAB to find the following.

(*i*) The average number of trains in the system

(*ii*) The average number of trains in the queue

(*iii*) Mean (or expected) waiting time in the queue (excluding service time)

(*iv*) Expected waiting time in the system (including service time)

5. The probability of *n* customer in the queue of (M/M/1:c/FCFS/∞) queue.

$$p_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \text{for } 0 \le n \le N \text{ and } \rho \ne 1 \\ \dfrac{1}{N+1}, & \text{for } 0 \le n \le N \text{ and } \rho = 1 \end{cases}$$

$$\rho = \lambda / \mu$$

Expected line length, i.e. the average number of customers in the system

$$L_s = \frac{\rho[1-(N+1)\rho^N + N\,\rho^{N+1}}{(1-\rho)(1-\rho^{N+1})},\ \rho \neq 1$$

Average number of customers in the queue

$$L_q = L_s - 1 + p_0 = \frac{\rho^2[1-N\rho^{N-1}+(N-1)\rho^N]}{(1-\rho)(1-\rho^{N+1})},\ \rho \neq 1$$

Waiting time in the queue

$$W_q = \frac{\rho^2[1-N\,\rho^{N-1}+(N-1)\rho^N]}{\lambda(1-\rho)(1-\rho^N)}$$

Waiting time in the system

$$W_s = W_q + \frac{1}{\mu}$$

Effective arrival rate

$$\lambda_{eff} = \lambda(1-p_N)$$

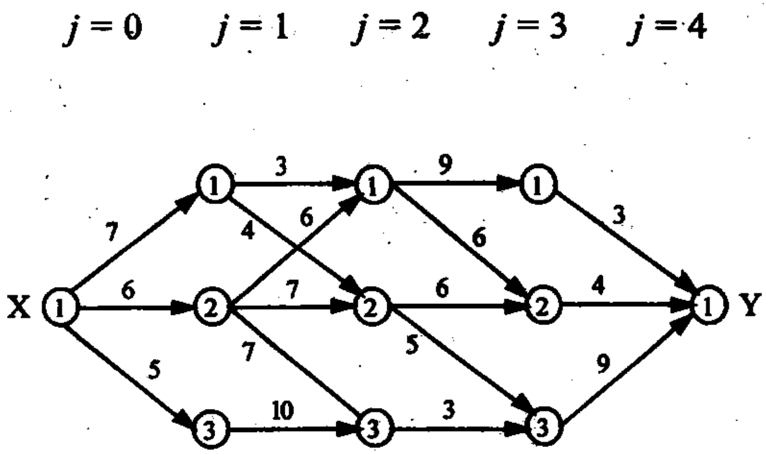
Patients arrive at a clinic according to a Poisson distribution at the rate of 50 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with a mean rate of 20 per hour.

(*i*) Find the effective arrival rate at the clinic.

(*ii*) What is the average number of customers in the queue ?

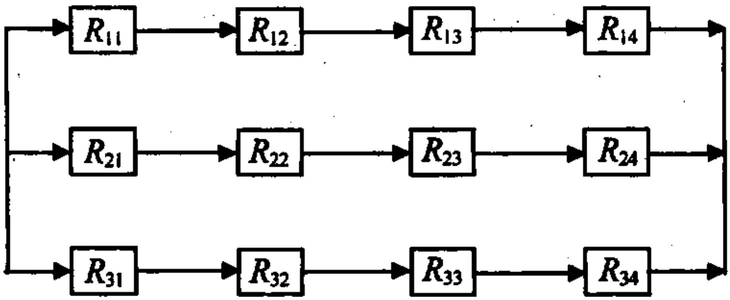
(*iii*) What is the expected waiting time until a patient is discharged from the clinic ?

6. Write MATLAB LINGO code to find the shortest path using dynamic programming method and the corresponding distance from the vertex X to the vertex Y along the edges joining various vertices lying between X and Y shown in the following figure. The numbers associated with the edges represent edge weights.



7. Write MATLAB code to estimate the probability to obtain 8 or more heads, if a coin is tossed 10 times, using the Monte Carlo simulation technique.

8. Consider the following diagram of a system of components



The reliability of the component R_{ij} is given by $R_{ij}(t) = e^{-ij\,\lambda t+1}$.

Write MATLAB LINGO code to calculate the system's reliability for 110 hours for $\lambda = 0.0045$.

9. The system comprises 400 transistors, 9,500 resistors, and 700 capacitors connected in series. The failure rates for these components are as follows :

Transistors have a failure rate of 0.9×10^{-7} per hour,

Resistors have a failure rate of 0.3×10^{-6} per hour, and

Capacitors have a failure rate of 0.5×10^{-6} per hour.

Write MATLAB code to find system's failure rate and reliability over 110 hours.

10. Field visit 5

11. Lab notebook & viva. 5