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A New Approach for Solving Transportation Problems with Mixed Constraints

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ABSTRACT

A new method based on transportation algorithm is proposed for finding a optimal more-for-less (MFL) solution for transportation problems with mixed constraints. The optimal MFL solution procedure is illustrated with numerical example. The proposed method is very simple, easy to understand and apply. The existence of a more-for-less situation in distribution problems provides useful information to a manager in deciding on the plant in which capacities are to be increased, and to assess the markets in which it is worthwhile to pursue efforts to increase demand.

1. Introduction

The transportation problem (TP) is a special class of linear programming problem, which deals with shipping commodities from sources to destinations. The objective of the TP is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. The TP finds application in industry, planning, communication network, scheduling, transportation and allotment etc. In literature, a good amount of research [2,5,7,17] is available to obtain an optimal solution for TPs with equality constraints. In real life, most of the TPs have mixed constraints, accommodating many applications not only in the distribution problems but also, in job scheduling, production inventory and investment analysis. The TPs with mixed constraints are not addressed in the literature because of the rigour required to solve these problems optimally. A literature search revealed no systematic method for finding an optimal solution for TPs with mixed constraints.

The MFL paradox in a TP occurs when it is possible to ship more 'total goods' for less (or equal) 'total cost' while shipping the same amount or more from each origin and to each destination , keeping all shipping costs non-negative. The occurrence of MFL in distribution problems is observed in nature. The existing literature [1, 3, 4, 6, 10-16] has demonstrated the identifying cases where MFL paradoxical situation exists and also, has provided various methods for finding MFL solution for transportation problems. Gupta et al.[4] and Arsham [1] obtained the more-for-less solution for the TPs with mixed constraints by relaxing the constraints and by introducing new slack variables. While yielding the best more-for-less solution, their method is tedious since it introduces more variables and requires solving sets of complex equations. The perturbed method was used for solving the TPs with constraints in [8,6,9]. Adlakha et al.[15] proposed a heuristic method for solving TPs with mixed constraints which is based on the theory of shadow price. In the heuristic algorithm for an MFL solution in Adlakha et al.[15], Vogel Approximation Method (VAM) and MODI (Modified Distribution) method were used. The primary goal of the MFL method is to minimize the total cost and not merely maximize the shipment load transported.

In this paper, we propose a new method for finding an optimal MFL solution of TPs with mixed constraints. The optimal MFL solution procedure is illustrated with the help of numerical example. The proposed method is very simple, easy to understand and apply. The MFL situation exists in reality and it could present managers with an opportunity for shipping more units for less or the same cost. The more-for-less analysis could be useful for managers in making important decisions such as increasing warehouse/plant capacity, or advertising efforts to increase demand at certain markets.

2. Transportation problem with mixed constraints

Consider the mathematical model for a TP with mixed constraints .

(P) Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} \geq a_i , \ i \in U \quad (1)$$

$$\sum_{j=1}^{n} x_{ij} \leq a_i , \ i \in V \quad (2)$$

$$\sum_{j=1}^{n} x_{ij} = a_i , \ i \in W \quad (3)$$

$$\sum_{i=1}^{m} x_{ij} \geq b_j , \ j \in Q \quad (4)$$

$$\sum_{i=1}^{m} x_{ij} \leq b_j , \ j \in T \quad (5)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \ j \in S \quad (6)$$

$$x_{ij} \geq 0, \ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and integers} \quad (7)$$

where U, V and W are pairwise disjoint subsets of $\{1,2,3,...,m\}$ such that $U \cup V \cup W = \{1,2,3,...,m\}$; Q, T and S are pairwise disjoint subsets of $\{1,2,3,...,n\}$ such that $Q \cup T \cup S = \{1,2,3,...,n\}$; c_{ij} is the cost of shipping one unit from supply point i to the demand point j ; a_i is the supply at supply point i ; b_j is the demand at demand point j and x_{ij} is the number of units shipped from supply point i to demand point j.

Now, the LBP (least bound problem) for the problem (P) is given below.

(LBP) Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \ i \in U$$

$$\sum_{j=1}^{n} x_{ij} = 0, \ i \in V$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \ i \in W$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \ j \in Q$$

$$\sum_{i=1}^{m} x_{ij} = 0, \ j \in T$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \ j \in S$$

$$x_{ij} \geq 0, \ i = 1,2,...,m \text{ and } j = 1,2,...,n \text{ and integers .}$$

Asharm [1] proved that the existence of a MFL situation in a regular TP requires only one condition namely, the existence of a location with negative plant-to-market shipping shadow price. The shadow prices are easily calculated from the solution of the TP with mixed constraints. The MFL solution is obtained from the optimal solution distribution by increasing and decreasing the shipping quantities while maintaining the minimum requirements for both supply and demand. The plant-to-market shipping shadow price (also called Modi index) at a cell (i, j) is

$u_i + v_j$ where u_i and v_j are shadow prices corresponding to the cell (i, j). The negative Modi index at a cell (i, j) indicates that we can increase the ith plant capacity / the demand of the jth market at the maximum possible level.

Theorem 1. The optimal MFL solution of a TP with mixed constraints is an optimal solution of a TP with mixed constraints which is obtained from the given TP with mixed constraints by changing the sign of columns and rows having negative Modi indices from $\leq$ to $=$ and $=$ to $\geq$.

Proof: Since the existence of a MFL situation in a TP requires only one condition namely, the existence of a location with negative Modi index. Therefore, the negative Modi index at a cell (i, j), $u_i + v_j$ indicates that we can achieve the supply of the ith source / the demand of the jth destination at the maximum possible level. Construct a TP with mixed constraints obtained from the given problem by changing the sign of columns and rows having negative Modi indices from $\leq$ to $=$ and $=$ to $\geq$ in the given problem. The newly constructed TP with mixed constraints is a TP problem with mixed constraints such that all the columns and rows having negative Modi indices can be achieved at the maximum level. Therefore, any solution of the newly constructed TP with mixed constraints is an MFL solution to the given problem. Thus, the optimal solution of the newly constructed TP with mixed constraints is an optimal MFL solution to the given TP with mixed constraints.
Hence the theorem.

Optimal MFL procedure:

We use the following procedure for finding an optimal MFL solution to a TP with mixed constraints.

Step 1. Form the LBP which is obtained from TP with mixed constraints by changing all inequalities to equalities with the lowest possible feasible right-hand side values.

Step 2. Balance LBP and find an optimal solution of the balanced LBP using the transportation algorithm.

Step 3. Place the load(s) of the dummy row(s)/ column(s) of the balanced LBP at the lowest cost feasible cells of the given TP to obtain a solution for the TP with mixed constraints.

Step 4. Create the Modi index matrix using the solution of the given TP obtained in the Step 3.

Step 5. Identify negative Modi indices and related columns and rows. If none exist, this is an optimal solution to TP with mixed constraints (no MFL paradox is present). STOP.

Step 6. Form a new TP with mixed constraints by changing the sign of columns and rows having negative Modi indices from ≤ to = and = to ≥ in the given problem.

Step 7. Obtain a solution of the new TP with mixed constraints using the Step 1 to the Step 3.

Step 8. The optimal solution for the new TP with mixed constraints obtained from the Step 7 is an optimal MFL solution of the given TP with mixed constraints (by the Theorem 1.).

3. Numerical Example

The proposed method for finding an optimal MFL solution to a TP with mixed constraints is illustrated by the following example.

Example 1. Consider the following TP with mixed constraints.

	1	2	3	Supply
1	2	5	4	= 5
2	6	3	1	≥ 6
3	8	9	2	≤ 9
Demand	= 8	≥ 10	≤ 5	

Now, LBP for the given TP with mixed constraints is given below.

	1	2	3	Supply
1	2	5	4	= 5
2	6	3	1	= 6
3	8	9	2	= 0
Demand	= 8	= 10	= 0	

Now, the optimal solution of LBP is given below by the transportation algorithm.

	1	2	3	Supply
1	2 **5**	5	4	= 5
2	6	3 **6**	1 **0**	= 6

3	8	9	2 **0**	= 0
4	0 **3**	0 **4**	0	=7
Demand	= 8	= 10	= 0	

Using the step 3, we obtain the following solution for the given problem.

	1	2	3	Supply
1	**5**			= 5
2	**3**	**10**	**0**	≥ 6
3			**0**	≤ 9
Demand	= 8	≥ 10	≤ 5	

Therefore, the solution for the given problem is $x_{11} = 5$, $x_{21} = 3$, $x_{22} = 10$, $x_{23} = 0$ and $x_{33} = 0$ for a flow of 18 units with the total transportation cost is $58.

Now, the Modi index matrix for the optimal solution of given problem is given below.

	v_1	v_2	v_3	u_i
u_1	2	**-1**	**-3**	-4
u_2	6	**3**	**1**	0
u_3	7	**4**	**2**	1
v_j	6	3	1	

Since the first row and the second and third columns have negative Modi indices, we consider the following new TP with mixed constraints.

	1	2	3	Supply
1	2	5	4	≥ 5
2	6	3	1	≥ 6
3	8	9	2	≤ 9
Demand	≥ 8	≥ 10	= 5	

Now, LBP for the new TP with mixed constraints is given below.

	1	2	3	Supply
1	2	5	4	= 5
2	6	3	1	= 6
3	8	9	2	= 0
4	0	0	0	= 12
Demand	= 8	= 10	= 5	

Now, the optimal solution of LBP for the new TP with mixed constraints is given below by the transportation algorithm.

	1	2	3	Supply
1	2 **5**	5	4	= 5
2	6	3 1	1 **5**	= 6
3	8	9	2 **0**	= 0
4	0 **3**	0 **9**	0	=12
Demand	= 8	= 10	= 5	

Using the step 3, we obtain the following solution for the new TP with mixed constraints.

	1	2	3	Supply
1	**8**	**0**		≥ 5
2		**10**	**5**	≥ 6
3			**0**	≤ 9
Demand	≥ 8	≥ 10	= 5	

Now, the Modi index matrix for the solution of the new TP is given below.

	v_1	v_2	v_3	u_i
u_1	2	5	3	2
u_2	0	3	1	0
u_3	1	4	2	1
v_j	0	3	1	

Since all the Modi indices are positive, the current solution is an optimal solution of the new TP with mixed constraints . Thus, by the Theorem 1., the optimal MFL solution for the given TP with mixed constraints is $x_{11} = 8$, $x_{12} = 0$, $x_{22} = 10$ $x_{23} = 5$ and $x_{33} = 0$ for a flow of 23 units with the total transportation cost is $51. The solution is better than the solution obtained by [4,15] because the shipping rate per unit is 2.22.

Note 1: For calculating Modi indices, we need $n + m - 1$ loading cells. So, we keep the cells that would be loaded using the zero point method even with a load of zero.

4. Conclusion

We have provided a new approach to obtain an optimal MFL solution of TPs with mixed constraints. The solution procedure for an optimal MFL solution of a TP with mixed constraints developed in this paper can also be used as an effective alternate solution method for solving certain transportation problems. The proposed method for an optimal MFL solution is very simple, easy to understand and apply. The MFL analysis could be useful for mangers in making strategic decisions such as increasing a ware-house stocking level or plant production capacity and advertising efforts to increase demand at certain markets. So, the new method for an optimal MFL solution using zero point method can serve managers by providing one of the best MFL solutions to a variety of distribution problems with mixed constraints.

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