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MHD Stagnation-Point Flow towards a Heated Stretching Sheet

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ABSTRACT

Boundary layer flow near a stagnation-point and heat transfer over a stretching sheet is very important due to its ever increasing wide range of applications. A similarity analysis is proposed to investigate the structure of the boundary layer near the stagnation-point region scaling group of transformation. A special form of Lie group transformations will be applied to find the similarity solution. The main advantage of this method is that no ad hoc assumptions or a prior knowledge of the equations under investigation is needed. Similarity solutions reduce the number of independent variables of the problem. As a result the governing non-linear partial differential equations reduce to non-linear ordinary differential equations. These equations are solved numerically using the Nactsheim-Swigert shooting iteration technique together with Runge-Kutta six order iteration scheme. Numerical results are obtained for the velocity and temperature. The obtained results are presented graphically and corresponding physical aspect of the problem are discussed.

Keywords: Scaling group of transformations; Stagnation-point flow; porous stretching sheet; Suction/injection; Heat source/Sink

1. Introduction

The problem of heat transfer in the boundary layer on a continuous moving surface has many practical applications in manufacturing processes in industry. The standard boundary layer equations play a control role in many aspects of fluid mechanics as they describe the motion of a slightly viscous fluid close to surface. Mechanics of non-linear fluids present a special challenge to engineers, physicists and the mathematicians. The non-linearity can manifest itself into a variety of ways.The boundary layer equations are especially interesting from a physical point of view because they have the capacity to admit a large number of invariant solutions i.e. basically closed-form solutions.

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Nomenclature

F	non-dimensional stream function
F^*	variable
$F^/$	streamwise velocity
S	suction parameter
M	Hartmann number
m	parameter
Cp	specific heat
B	Magnetic parameter
G	absolute invariant, $x^r\ \varphi^*$
Pr	Prandtl number
p, q	Variables
T	temperature of the fluid
T_w	temperature of the wall
T_∞	free stream temperature
u, v	Components of velocity in the x and y-directions
Greek Symbols	
$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha^/, \alpha^{//}$	transformation parameters
$\beta^/, \beta^{//}$	transformation parameters
η	similarity variable
Γ	Lie group transformations
κ	coefficient of thermal diffusivity
μ	dynamic viscosity
μ^*	reference viscosity
v	kinematic viscosity
φ	stream function
φ^*	variable
ρ	density of the fluid
θ	Non-dimensional temperature

In the present context, invariant solutions are meant to be a reduction to a simpler equation such as an ordinary differential equation (ODE). Prandtl's boundary layer equations admit more and different symmetry groups or simply symmetries are invariant transformations which do not alter the structural form of the equation underinvestigation Bluman and Kumei (1989). The main advantage of the symmetry method is that it can be applied successfully to non-linear differential equations governing the motion of viscous fluid. Lie-group analysis was named after sophius Lie who developed it to find point transformations which map a given differential equation to itself. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations and no ad hoc assumptions or a prior knowledge of the equation under investigation is needed. The differential equation remains invariant under some continuous group of transformations usually known as symmetries of a differential equation. Actually a symmetry group maps any solution to another solution. In case of the scaling group of transformations, the group-invariant solutions are none but the well known similarity solutions Pakdemiril and Yurrusoy (1998). Similarity solutions are very useful in the sense that they reduce the independent variables of the problem. In this paper, we apply a special form of Lie-group transformations to the problem of stagnation-point flow and heat transfer over a porous stretching sheet. It is well known that the flow in a boundary layer separates in the regions of adverse pressure gradient and the occurrence of separation has several undesirable effects in so far as it leads to increase in the drag on the body immersed in the flow and adversely affects the heat transfer from the surface of the body. Separation can be prevented by suction as the low-energy fluid in the boundary layer is removed. In fact, suction tends to stabilize the boundary layer flow. On the country, the wall shear stress and hence the friction drag is reduced by blowing. The stability of the boundary layer and transition to turbulence are also significantly influenced by continuous suction and blowing.

The heat, mass and momentum transfer in the laminar boundary layer flow on a stretching sheet are important from theoretical as well as practical point of view because of their wider applications to polymer technology and metallurgy. In the area of the steady flow of an incompressible fluid over an infinite porous plate subject to suction or blowing various aspects of the problem have been investigated by many authors. Suction and blowing in convective heat transfer over a vertical permeable surface embedded in a porous medium was analysis by Cheng (1977).

2. Equation of motion

We consider the two-dimensional steady flow of an incompressible, viscous liquid near a stagnation-point at a porous surface coinciding with the plane $y = 0$, the flow being confined to $y > 0$. We introduce two equal and oppositeforces along the x-axis so that the wall is stretched keeping the origin fixed.

The boundary layer equations for steady two-dimensional stagnation-point flow over a heated porous stretching surface are, in the usual notations.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + v\frac{\partial^2 u}{\partial y^2} + \frac{\sigma B_0^2}{\rho}(U - u) \tag{2}$$

In Equation (2), $U(x)$ stands for the stagnation-point velocity in the inviscid free stream, u and v are the components of velocity respectively in the x and y directions, μ is the coefficient of fluid viscosity, ρ is the fluid density, $v = \frac{\mu}{\rho}$ is the kinematic viscosity, By using the boundary layer approximations and neglecting viscous dissipation, the equation for temperature T in presence of heat source or heat sink is

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where T is the temperature, k is the coefficient of thermal of diffusivity of the fluid.

3. Boundary Conditions

The appropriate boundary conditions for the problem are given by

$$u=cx, v=v_w, T=T_w \ at\ y=0, \tag{4}$$

$$u\to U(x)=ax, T\to T_\infty \ as\ \ y\to\infty \tag{5}$$

Here $c(>0)$ and $a(>0)$ are constants, v_w is the velocity, T_w is the uniform wall temperature, T_∞ is the free stream temperature, T_w and T_∞ are also constants with $T_w > T_\infty$.

4. Method of solution

We now introduce the following relations for u, v and θ as

$$u=\frac{\partial\varphi}{\partial y}, v=-\frac{\partial\varphi}{\partial x}\ and\ \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

where φ is the stream function.

Using the relations (6) in the boundary layer Equation (2) and in the energy Equation (3) we get the following equations

$$\frac{\partial\varphi}{\partial y}\frac{\partial^2\varphi}{\partial x\partial y}-\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial y2}=U\frac{\partial U}{\partial x}+v\frac{\partial^3\varphi}{\partial y^3}+\frac{\sigma B_0^2}{\rho}(U-\frac{\partial\varphi}{\partial y}) \tag{7}$$

and

$$\frac{\partial\varphi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\varphi}{\partial x}\frac{\partial\theta}{\partial y}=k\frac{\partial^2\theta}{\partial y^2} \tag{8}$$

The boundary conditions (4) and (5) then become

$$\frac{\partial\varphi}{\partial y}=cx, \frac{\partial\varphi}{\partial x}=-v_w, \theta=1\ at\ y=0, \tag{9}$$

$$\frac{\partial\varphi}{\partial y}\to U(x)=ax, \theta\to 0\ as\ y\to\infty. \tag{10}$$

5. Scaling group of transformations

We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations (Tapanidis et al.2003).

$\Gamma: x^* = x\, e^{\varepsilon\alpha_1}, \quad y^* = y\, e^{\varepsilon\alpha_2}, \quad \varphi^* = \varphi e^{\varepsilon\alpha_3}, \quad u^* = u e^{\varepsilon\alpha_4}, \quad v^* = v\, e^{\varepsilon\alpha_5},$

$$U^* = U e^{\varepsilon\alpha_6}, \quad \theta^* = \theta\, e^{\varepsilon\alpha_7} \qquad (11)$$

Eq.(11) may be considered as a point-transformation which transforms co-ordinates $(x, y, \varphi, u, v, U, \theta)$ to the coordinates $x^*, y^*, \varphi^*, u^*, v^*, U^*, \theta^*)$.

Substituting Equation (11) in Equations (7) and (8) we get,

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\varphi^*}{\partial y^*}\frac{\partial^2\varphi^*}{\partial x^*\partial y^*} - \frac{\partial\varphi^*}{\partial x^*}\frac{\partial^2\varphi}{\partial y^{*2}}\right) = e^{\varepsilon(\alpha_1-2\alpha_6)}U^*\frac{\partial U^*}{\partial x^*} + \nu e^{\varepsilon(3\alpha_2-\alpha_3)}\frac{\partial^3\varphi^*}{\partial y^{*3}} + \frac{\sigma B_0^2}{\rho}e^{-\epsilon\alpha_6}U^* - \frac{\sigma B_0^2}{\rho}e^{\epsilon(\alpha_2-\alpha_3)}\frac{\partial\varphi^*}{\partial y^*} \qquad (12)$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_7)}\left(\frac{\partial\varphi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*} - \frac{\partial\varphi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}\right) = k e^{\varepsilon(2\alpha_2-\alpha_7)}\frac{\partial^2\theta^*}{\partial y^*} \qquad (13)$$

The system will remain invariant under the group of transformations Γ, so we would have the following relations among the parameters, namely

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = \alpha_1 - 2\alpha_6 \Rightarrow \alpha_2 - \alpha_3 + \alpha_6 = 0 \qquad (i)$$

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 \Rightarrow \alpha_1 - \alpha_2 - \alpha_3 = 0 \qquad (ii)$$

Now adding (i) and (ii) we get, $\alpha_1 - 2\alpha_3 + \alpha_6 = 0$

$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_7 = 2\alpha_2 - \alpha_7 \Rightarrow \; \alpha_1 - \alpha_2 - \alpha_3 = 0$

from the boundary conditions

$e^{\varepsilon(\alpha_2-\alpha_3)}\frac{\partial\varphi^*}{\partial y^*} = e^{-\varepsilon\alpha_1}cx^*, e^{(\alpha_1-\alpha_3)}\frac{\partial\varphi^*}{\partial x^*} = -v_w, e^{-\varepsilon\alpha_7}\theta^* = 1$

we get, $\alpha_2 - \alpha_3 = -\alpha_1,\; \alpha_1 = \alpha_3, \alpha_7 = 0$

i.e. $\alpha_2 = 0 \qquad (iii)$

From (i), (ii) and (iii) we get, $\alpha_6 = \alpha_3$, $\alpha_1 = \alpha_3$ i.e. $\alpha_1 = \alpha_3 = \alpha_6$

$$\frac{\partial \varphi^*}{\partial y^*} = cx^* \ , \frac{\partial \varphi^*}{\partial x^*} = -v_w, \quad \theta^* = 1 \ at \ y^* = 0 \tag{14}$$

and

$$\frac{\partial \varphi^*}{\partial y^*} \to U^* = ax^* \ , \theta^* \to 0 \ as \ y^* \to \infty \tag{15}$$

with the additional conditions $\alpha_1 = \alpha_3 = \alpha_4 = \alpha_6, \alpha_2 = \alpha_5 = \alpha_7 = 0$. thus the set Γ reduces to a one parameter group of transformations as

$$x^* = xe^{\varepsilon\alpha_1}, y^* = y, \varphi^* = \varphi e^{\varepsilon\alpha_1}, u^* = ue^{\varepsilon\alpha_1}, U^* = Ue^{\varepsilon\alpha_1}, \theta^* = \theta. \tag{16}$$

First we derive an absolute invariant which is a function of the dependent variable, namely $\eta = yx^*$. For this purpose, we write

$$x^* = Bx, B = e^{\varepsilon\alpha_1}, y^* = B^{\frac{\alpha_2}{\alpha_1}}y, \ \ \varphi^* = B^{\frac{\alpha_3}{\alpha_1}}\varphi, \qquad U^* = UB^{\frac{\alpha_6}{\alpha_1}}. \tag{17}$$

To establish $y^*x^{*s} = yx^s$ we have $y^*x^{*s} = yB^{\frac{\alpha_2}{\alpha_1}}B^s x^s = yx^s B^{s+\frac{\alpha_2}{\alpha_1}}$. putting $s + \frac{\alpha_2}{\alpha_1} = 0$ we get, $y^*x^{*s} = yx^s$. since $\alpha_2 = 0$ so $s = 0$ and we get $\eta = y^*$. Thus we obtain

$$\eta = y^* \tag{18}$$

as an absolute invariant. We now find a second absolute invariant G, which involves the dependent variable φ. Let us assume that $G = x^r\varphi$. We will find r such that $x^r\varphi = x^{*r}\varphi^*$. Now $x^{*r}\varphi^* = B^r x^r B^{\frac{\alpha_3}{\alpha_1}}\varphi = B^{r+\frac{\alpha_3}{\alpha_1}}\varphi x^r$. By putting $r + \frac{\alpha_3}{\alpha_1} = 0$ we get, $r = -\frac{\alpha_3}{\alpha_1} = -1$ $(since \ \alpha_1 = \alpha_3)$. Thus we get the second absolute invariant $G \ as \ G = {x^*}^{-1}\varphi^*$.

Putting

$$G = F(\eta). \tag{19}$$

we also have $\theta^* = \theta(\eta)$.

In view of the relations (18) and (19), Equations. (12) and (13) become

$$F^{/2} - FF^{//} = a^2 + vF^{///} + \frac{\sigma B_0^2}{\rho}(a - F) \tag{20}$$

and

$$F\theta^{/} + k\theta^{//} = 0 \tag{21}$$

The boundary conditions are transformed are transformed to

$$F^{/}(\eta) = c.F(\eta) = -v_w \ and \ \theta(\eta) = 1 \ at \ \ \eta = \tag{22}$$

$$F^{/}(\eta) \to a, \quad \theta(\eta) \to 0 \ as \ \eta \to \infty. \tag{23}$$

Again we introduce the following the transformations for η, F and θ in Eqs.(20) and (21).

$$\eta = v^{\alpha} c^{\beta} \eta^*, F = v^{\alpha^{/}} c^{\beta^{/}} F^*, \theta = v^{\alpha^{//}} c^{\beta^{//}} \bar{\theta}. \tag{24}$$

Taking $F^* = f$ and $\bar{\theta} = \theta$ the Equation (20) and (21) finally take the following form:

$$f^{///} + ff^{//} - f^{/2} + \frac{a^2}{c^2} + \frac{\sigma B_0^2}{\rho}(\frac{a}{c} - f^{-}) = 0 \tag{25}$$

and

$$\theta^{//} + \Pr\left(f\theta^{/}\right) = 0 \tag{26}$$

where $k_1 = \frac{v}{k_c}$ is the permeability parameter of the porous medium, $Pr = \frac{v}{k}$ is the prandtl number and $\lambda = \frac{Q_0}{\rho^c p^c}$ is the heat source ($\lambda > 0$) or sink ($\lambda < 0$) parameter.

The boundary conditions take the following forms

$$f^{/} = 1, \ f = S, \ \theta = 1 \ at \ \eta^* = 0 \tag{27}$$

and

$$f^{/} \to \frac{a}{c}, \qquad \theta \to 0 \ as \ \eta^* \to \infty. \tag{28}$$

where $S = \frac{v_w}{\sqrt{\nu c}}$ $S > 0$ $(i.e. v_w < 0)$ corresponds to suction and $S < 0$ $(i.e. v_w > 0)$ corresponds to blowing.

6. Numerical method for solution

The above Equations (25) and (26) along with boundary conditions are solved by converting them to an initial value problem

$$f^{/} = z,\ z^{/} = p, p^{/} = z^2 - fp - \frac{a^2}{c^2} - \frac{\sigma B_0^2}{\rho}(\frac{a}{c} - z) \qquad (29)$$

$$\theta^{/} = q, q^{/} = -\Pr(fq) \qquad (30)$$

With the boundary conditions

$$f(0) = S, \qquad f^{/}(0) = 1, \theta(0) = 1 \qquad (31)$$

In order to integrate Equations (29) and (30) as an initial value problem we require a value for $p(0)$ i.e. $f^{//}(0)$ and $q(0)$ i.e.$\theta^{/}(0)$ but no such values are given in the boundary. The suitable guess values for $f^{//}(0)$ and $\theta^{/}(0)$ are chosen and then integration is carried out by Nactsheim-Swigert shooting iteration technique together with Runge-Kutta six order iteration scheme. We compare the calculated values for $f^{/}$ and θ at $\eta = 5$ with the given boundary conditions $f^{/}(5) = a/c$ and $\theta(5) = 0$ and adjust the estimated values, $f^{//}(0)$ and $\theta^{/}(0)$, to give a better approximation for the solution. There are two unknown surface conditions values for $f^{//}(0)$ and $\theta^{/}(0)$, as there are two asymptotic boundary conditions.

7. Results and discussions

In order to analyze the results, numerical computations have been carried out using the method described in the previous section for various values of the suction parameter S, magnetic parameter M, parameter m, and the prandtl number Pr. For illustrations of the results, numerical values are plotted in Fig 3.1-3.7. First, we present the result for the variation of the magnetic parameter M. in Fig.3.1 streamwise velocity profiles are shown for different values of $M(M = 0, 0.5, 1)$ with $m = 0.1, S = 1$ and $Pr = 0.05$ the streamwise velocity curves show that the rate of transport decreases with the increase in M. it clearly indicates that the transverse magnetic field opposes the transport phenomena. This is due to the fact that variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to the transport phenomena. In all cases the velocity vanishes at some large distance from the sheet Fig.3.3 exhibits the temperature profiles for different values of $M(M = 0, 0.5, 1)$. in each case, temperature asymptotically decreases with the increase of η. But the temperature and heat transfer rate are found to increase with the increase of M. Now we focus on the velocity and temperature distribution for the variation of suction parameter S in the absence of a magnetic field $(M = 0)$ when $m = 0.5$ and $\Pr = 0.05$. with the increasing S, the streamwise velocity is found to decrease (Fig.) i.e. suction causes a decrease in the velocity of fluid in the boundary layer region. The physical explanation for such a behavior is as follows. In the case of suction, the heated fluid is pushed toward the wall where the buoyancy forces can act to retard the fluid due to the high influence of the viscosity. This effect acts to decrease the wall shear stress .Fig shows that the temperature $\theta(\eta)$ in the boundary layer also decrease with the increasing of suction

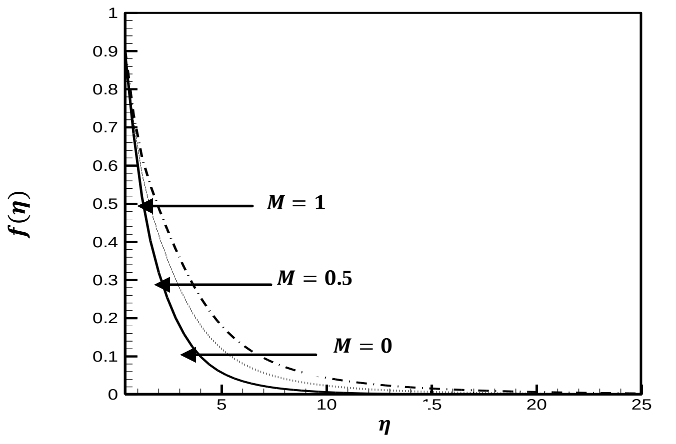

Figure 1. Variation of stream-wise velocity $f^{/}(\eta)$ with η for several values of M when $Pr = 0.05, m = 0.1$ and $S = 1$

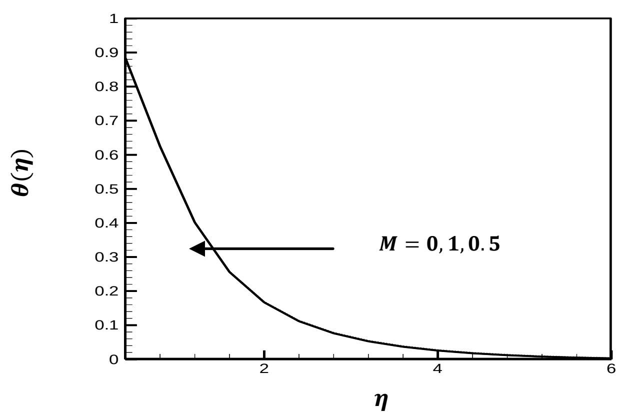

Figure 2. Variation of temperature $\theta(\eta)$ with η for several values of M when $Pr = 0.05, m = 0.1$ and $S = 1$

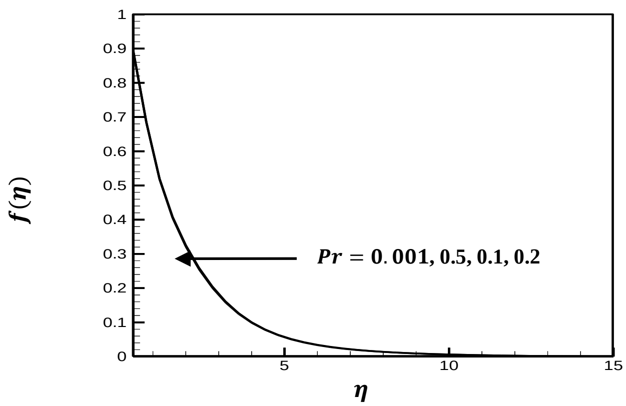

Figure 3. Variation of stream-wise velocity $f^{/}(\eta)$ with η for several values of Pr when $M = 1, m = -0.5$ and $S = 1$

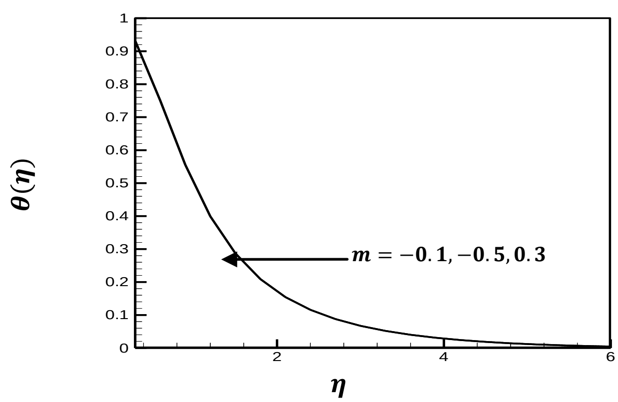

Figure 4. Variation of temperature $\theta(\eta)$ with η for several values of m when $Pr = 0.05, S = 0$ and $M = 1$

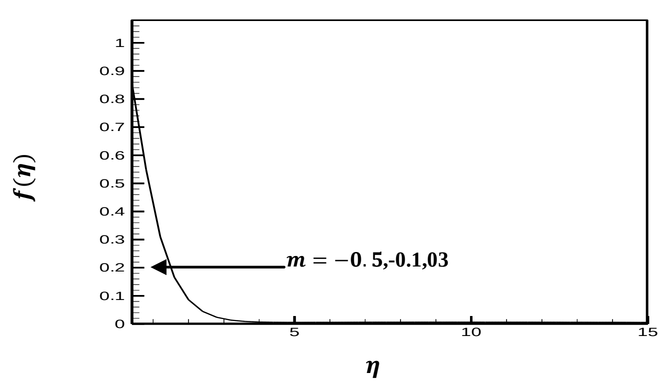

Figure 5. Variation of stream-wise velocity $f^{/}(\eta)$ with η for several values of m when $Pr = 0.05, S = 1$ and $M = 1$

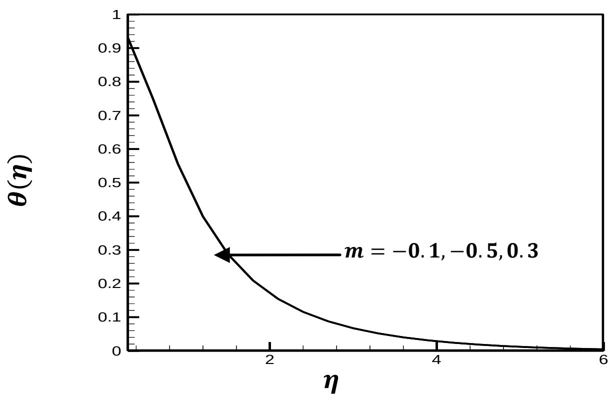

Figure 6. Variation of temperature $\theta(\eta)$ with η for several values of m when $Pr = 0.05, S = 0$ and $M = 1$

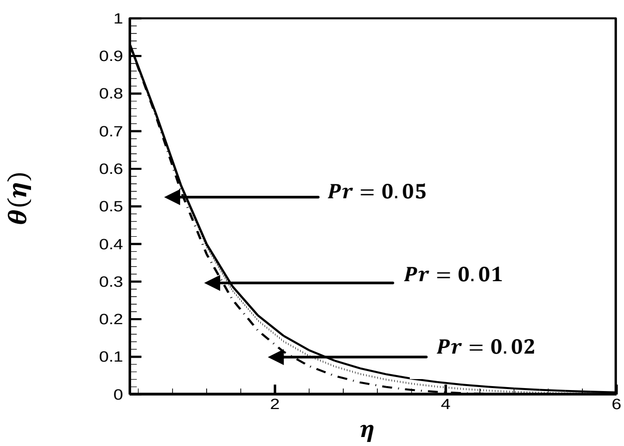

Figure 7. Variation of temperature $\theta(\eta)$ with η for several values of Pr when $m = -0.1, S = 0$ and $M = 1$

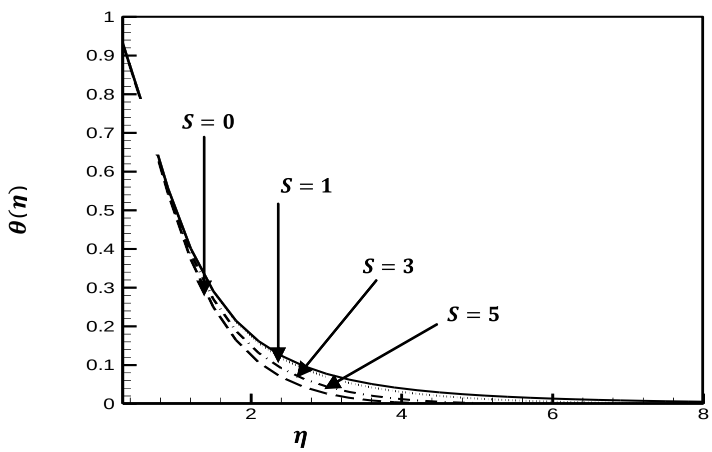

Figure 8. Variation of temperature $\theta(\eta)$ with η for several values of S when $Pr = 0.05, m = 0.5$ and $M = 1$

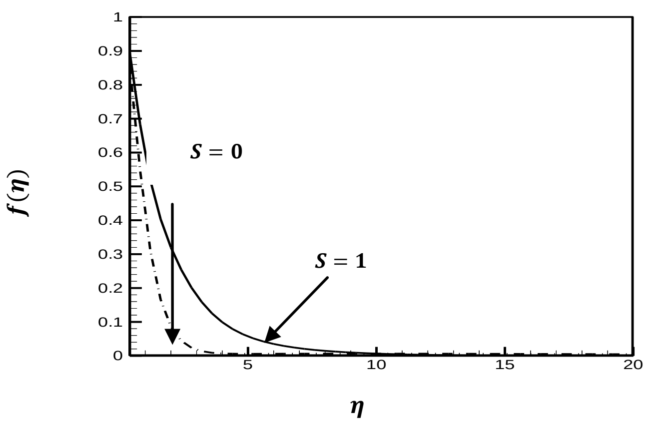

Figure 9. Variation of stream-wise velocity $f^{/}(\eta)$ with η for several values of s when $Pr = 0.05, m = 0.5$ and $M = 1$.

8. Conclusion

The present study gives the solutions for steady combined convective boundary layer flow and heat transfer over an elastic surface with power law stretching velocity in the presence of a transverse magnetic field. The effect of a transverse magnetic field on a viscous incompressible conducting fluid is to suppress the velocity field, which in turn causes the enhancement of the temperature field. Results pertaining to the present study indicate that due to suction, the skin friction decreases while the rate of heat transfer increases. The temperature in the boundary layer decreases due to suction. The stream-wise velocity decreases with an increase in m (power law index parameter), but it has no significant effect on temperature. The thermal boundary layer thickness decreases with the increase of Pr.

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