Journal of Physical Sciences, Vol. 17, 2013, 77-86
ISSN: 0972-8791, www.vidyasagar.ac.in/journal
Published on 26 December 2013

A New Method for Ranking Exponential Fuzzy Numbers with use Weighted Average and Weighted Width in TRD Distance

S. Rezvani*, M. Molani and M. Ebrahimi

Department of Industrial Engineering, Ayatollah Amoli Branch, Islamic Azad University, Amol, Iran
Email: Salim_rezvani@yahoo.com

Received 26 February 2013; accepted 30 September 2013

ABSTRACT

In this paper, we want to present a new method for ranking exponential fuzzy numbers with use weighted average and weighted width in TRD distance. Weighted average and weighted width of exponential fuzzy numbers combines in TRD distance for ranking method. A simpler and easier approach is proposed for the ranking of generalized trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

Keywords: Exponential fuzzy numbers, TRD distance, weighted average, weighted width.

1. Introduction

Ranking fuzzy numbers are an important tool in decision making. In fuzzy decision analysis, fuzzy quantities are used to describe the performance of alternatives in modeling a real-world problem. Various ranking procedures have been developed since 1976 when the theory of fuzzy sets were first introduced by Zadeh [27]. Ranking fuzzy numbers were first proposed by Jain [13] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Since then, various procedures to rank fuzzy quantities are proposed by various researchers. Bortolan and Degani [4] reviewed some of these ranking methods [3,13,18,19] for ranking fuzzy subsets. Chen [6] presented ranking fuzzy numbers with maximizing set and minimizing set. Dubois and Prade [12] presented the mean value of a fuzzy number. Lee and Li [16] presented a comparison of fuzzy numbers based on the probability measure of fuzzy events. Delgado et al. [11] presented a procedure for ranking fuzzy numbers. Campos and Munoz [5] presented a subjective approach for ranking fuzzy numbers. Kim and Park [14] presented a method of ranking fuzzy numbers with index of optimism. [17] and Wang and Lee [25] also used the centroid concept in developing their ranking index. Chen and Chen [7] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α -levels of trapezoidal fuzzy numbers. Chen and Chen [8] presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Also, Allahviranloo [2] proposed a method foe ranking of fuzzy numbers using new weighted distance. Rezvani ([18],[23]) evaluated the system of ranking fuzzy numbers. Moreover, Rezvani [22] proposed a new method for ranking in areas of two generalized trapezoidal fuzzy numbers.

In this paper, we want to present a new method for ranking exponential fuzzy numbers with use weighted average and weighted width in TRD distance. Weighted average and weighted width of exponential fuzzy numbers combines in TRD distance for ranking method. A simpler and easier approach is proposed for the ranking of generalized trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

2. Preliminaries

Definition 1. Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line *R*, whose membership function μ_A satisfies the following conditions,

i) μ_{A} is a continuous mapping from R to the closed interval $[0,1]$,

ii) $\mu_{\mathrm{A}}(x) = 0$, $-\infty < u \le c$,

iii) $\mu_{\mathrm{A}}(x) = L(x)$ is strictly increasing on $[c, a]$,

iv) $\mu_{\mathrm{A}}(x) = w$, $a \le x \le b$,

v) $\mu_{\mathrm{A}}(x) = R(x)$ is strictly increasing on $[b, d]$,

vi) $\mu_{\mathrm{A}}(x) = 0$, $d \le x < \infty$

where $0 < w \le 1$ and *a,b,c* and d are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by

$$A = (c, a, b, d; w)_{LR} \quad .$$

When $w = 1$, this type of generalized fuzzy number is called normal fuzzy number and is represented by $A = (c, a, b, d)_{LR}$.

However, these fuzzy numbers always have a fix range as $[c, d]$. Here, we define its general from as follows:

$$f_{\mathrm{A}}(x) = \begin{cases} we^{-[(a-x)/\gamma]} & x \le a \\ w & a \le x \le b \\ we^{-[(a-b)/\beta]} \quad if & b \le x \end{cases} \tag{1}$$

where $0 < w \le 1$ and a, b are real numbers and γ, β are positive real numbers. We denote this type of generalized exponential fuzzy number as $A = (a, b, \gamma, \beta; w)_E$. Especially, when *w = 1*, we denote it as $A = (a, b, \gamma, \beta)_E$.

We define the representation of generalized exponential fuzzy number based on the integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number $A = (a, b, \gamma, \beta; w)_E$, where $0 < w \le 1$ and a, b are real numbers *formula (1).* Now, let two monotonic functions be

$$L(x) = we^{-[(a-x)/\gamma]}, \qquad R(x) = we^{-[(a-b)/\beta]} \tag{2}$$

Then the inverse functions of function *L* and *R* are L^{-1} and R^{-1} respectively.

$$L^{-1}(h) = a - \gamma(\ln\frac{w}{h}), \tag{3}$$

$$L^{-1}(h) = b + \beta(\ln\frac{w}{h}). \tag{4}$$

Definition 2. The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number *A*

$$I(A) = \int_0^1 [CL_A(\alpha) + (1-C)R_A(\alpha)]d\alpha \tag{5}$$

and

$$D(A) = \int_0^1 [R_A(\alpha) - L_A(\alpha)]f(\alpha)d\alpha \tag{6}$$

Here $0 \le C \le 1$ denotes an "optimism/pessimism" coefficient in conducting operations on fuzzy numbers. The function $f(\alpha)$ is nonnegative and increasing function on $[0, 1]$ with $f(0)=0, f(1)=1$ and $\int_0^1 f(\alpha)d\alpha = \frac{1}{2}$. The function $f(\alpha)$ is also called weighting function. In actual applications, function $f(\alpha)$ can be chosen according to the actual situation. In this article, in practical case, we assume that $f(\alpha) = \alpha$.

3. Usage TRD Distance in Ranking Exponential Fuzzy Numbers

In this section some important results, that are useful for the proposed approach, are proved.

Theorem 1. The values constitute of weighted averaged representative and weighted width, of the exponential fuzzy number *A* are following

$$I(A) = w[C\gamma(e^{\frac{1-a}{\gamma}} - e^{\frac{-a}{\gamma}})] + w[(1-C)\beta(e^{\frac{b}{\beta}} - e^{\frac{b-1}{\beta}})] \tag{7}$$

and

$$D(A) = w[\beta^2 e^{\frac{b}{\beta}} - e^{\frac{b-1}{\beta}}(\beta+\beta^2)] - w[\gamma^2 e^{\frac{-a}{\gamma}} + e^{\frac{1-a}{\gamma}}(\gamma-\gamma^2)] \tag{8}$$

Proof:

$$I(A) = \int_0^1 [CL_A(\alpha) + (1-C)R_A(\alpha)]d\alpha = \int_0^1 [Cwe^{-[(a-\alpha)/\gamma]} + (1-C)we^{-[(a-b)/\beta]}]d\alpha$$

$$= Cw\gamma[e^{\frac{1-a}{\gamma}} - e^{\frac{-a}{\gamma}}] + (1-C)w\beta[e^{\frac{b}{\beta}} - e^{\frac{b-1}{\beta}}] = w[C\gamma(e^{\frac{1-a}{\gamma}} - e^{\frac{-a}{\gamma}})] +$$

$$w[(1-C)\beta(e^{\frac{b}{\beta}} - e^{\frac{b-1}{\beta}})]$$

and

$$D(A) = \int_0^1 [R_A(\alpha) - L_A(\alpha)]f(\alpha)d\alpha = \int_0^1 [we^{-[(a-b)/\beta]} - we^{-[(a-x)/\gamma]}]f(\alpha)d\alpha$$

$$= \int_0^1 [we^{-[(a-b)/\beta]} - we^{-[(a-x)/\gamma]}]\alpha d\alpha = w[(-\beta e^{\frac{b-1}{\beta}} - \beta^2 e^{\frac{b-1}{\beta}}) + \beta^2 e^{\frac{b}{\beta}}] -$$

$$w[(\gamma e^{\frac{1-a}{\gamma}} - \gamma^2 e^{\frac{1-a}{\gamma}}) + \gamma^2 e^{\frac{-a}{\gamma}}]$$

$$= w\,[\beta^2 e^{\frac{b}{\beta}} - e^{\frac{b-1}{\beta}}(\beta + \beta^2)] - w\,[\gamma^2 e^{\frac{-a}{\gamma}} + e^{\frac{1-a}{\gamma}}(\gamma - \gamma^2)]$$

Theorem 2. TRD distance between the exponential fuzzy number A as following

$$TRD(A) = \sqrt{[I(A)]^2 + [D(A)]^2} \tag{9}$$

Theorem 3. *TRD* distance between the exponential fuzzy numbers *A* and *B* as following

$$TRD(A,B) = \sqrt{[I(A) - I(B)]^2 + [D(A) - D(B)]^2} \tag{10}$$

Definition 3. Let $A_1 = (a_1, b_1, \alpha_1, \beta_1; w_1)_E$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2; w_2)_E$ be two generalized exponential fuzzy numbers, then

i) If $TRD(A_1) < TRD(A_2)$, then $A_1 < A_2$,

ii) If $TRD(A_1) > TRD(A_2)$, then $A_1 > A_2$,

iii) If $TRD(A_1) \sim TRD(A_2)$, then $A_1 \sim A_2$.

3.1. Method to Find the Values of TRD(A) and TRD(B)

Let $A_1 = (a_1, b_1, \alpha_1, \beta_1; w_1)_E$ and $A_2 = (a_2, b_2, \alpha_2, \beta_2; w_2)_E$ be two generalized exponential fuzzy numbers, then use the following steps to find the values of $TRD(A_1)$ and $TRD(A_2)$

* Step 1

Find $I(A_1)$ and $I(A_2)$

* Step 2

Find $D(A_1)$ and $D(A_2)$

* Step 3

Calculation $TRD(A_1)$ and $TRD(A_2)$ and use definition 3. , for ranking this method.

4. Validation of another Proposed Method

Example 1. Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 7)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$I(A) = 0.31C + 0.33, \quad I(B) = 3.6C + 0.42$$

* Step 2

$$D(A) = -0.14, \qquad D(B) = -2.8$$

* Step 3

$$TRD(A) = \sqrt{[0.31C + 0.33]^2 + [-0.14]^2}$$

For
$C = 0 \Rightarrow TRD(A) = 0.36$,
$C = 0.5 \Rightarrow TRD(A) = 0.5$,
$C = 1 \Rightarrow TRD(A) = 0.65$,

$$TRD(B) = \sqrt{[3.6C + 0.42]^2 + [-2.8]^2}$$

For
$C = 0 \Rightarrow TRD(B) = 2.83$,
$C = 0.5 \Rightarrow TRD(B) = 3.57$,
$C = 1 \Rightarrow TRD(B) = 4.9$,
Then, $TRD(A) < TRD(B) \Rightarrow A < B$.

Example 2. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$I(A) = 2.8C + 0.65, \quad I(B) = 4.91C + 0.84$$

* Step 2

$$D(A) = -1.03, \qquad D(B) = -3.98$$

* Step 3

$$TRD(A) = \sqrt{[2.8C + 0.65]^2 + [-1.03]^2}$$

For
$C = 0 \Rightarrow TRD(A) = 1.22$,
$C = 0.5 \Rightarrow TRD(A) = 2.29$,
$C = 1 \Rightarrow TRD(A) = 3.6$,

$$TRD(B) = \sqrt{[4.91C + 0.84]^2 + [-3.98]^2}$$

For
$C = 0 \Rightarrow TRD(B) = 4.07$,
$C = 0.5 \Rightarrow TRD(B) = 5.17$,
$C = 1 \Rightarrow TRD(B) = 6.99$,
Then, $TRD(A) < TRD(B) \Rightarrow A < B$.

Example 3. Let $A = (0.1, 0.2, 0.4, .5; 1)$ and $B = (1,1,1,1;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$I(A) = 2.8C + 0.65, \quad I(B) = -1.08C + 1.71$$

* Step 2

$$D(A) = -1.03, \qquad D(B) = -0.34$$

* Step 3

$$TRD(A)=\sqrt{[2.8C+0.65]^2+[-1.03]^2}$$

For
$C=0 \Rightarrow TRD(A)=1.22$,
$C=0.5 \Rightarrow TRD(A)=2.29$,
$C=1 \Rightarrow TRD(A)=3.6$,

$$TRD(B)=\sqrt{[-1.08C+1.71]^2+[0.34]^2}$$

For
$C=0 \Rightarrow TRD(B)=1.14$,
$C=0.5 \Rightarrow TRD(B)=1.22$,
$C=1 \Rightarrow TRD(B)=0.72$,
Then, $TRD(A)>TRD(B) \Rightarrow A>B$.

Example 4. Let $A=(-0.5,-0.3,-0.3,-0.1;1)$ and $B=(0.1,0.3,0.3,0.5;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$I(A)=-42518.8C+42518.83, \qquad I(B)=4.91C+0.84$$

* Step 2

$$D(A)=38268.9, \qquad D(B)=-3.98$$

* Step 3

$$TRD(A)=\sqrt{[-42518.8C+42518.83]^2+[38268.9]^2}$$

For
$C=0 \Rightarrow TRD(A)=57204.5$,
$C=0.5 \Rightarrow TRD(A)=43777.5$,
$C=1 \Rightarrow TRD(A)=38268.9$,

$$TRD(B)=\sqrt{[4.91C+0.84]^2+[-3.98]^2}$$

For
$C=0 \Rightarrow TRD(B)=4.07$,
$C=0.5 \Rightarrow TRD(B)=5.17$,
$C=1 \Rightarrow TRD(B)=6.99$,
Then, $TRD(A)>TRD(B) \Rightarrow A>B$.

Example 5. Let $A=(0.3,0.5,0.5,1;1)$ and $B=(0.1,0.6,0.6,0.8;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$I(A)=0.7C+1.04, \qquad I(B)=0.97C+1.2$$

* Step 2

$$D(A)=-0.71, \qquad D(B)=-0.89$$

* Step 3

$$TRD(A)=\sqrt{[0.7C+1.04]^2+[-0.71]^2}$$

For
$C=0 \Rightarrow TRD(A)=1.28$,
$C=0.5 \Rightarrow TRD(A)=1.58$,
$C=1 \Rightarrow TRD(A)=1.89$,

$$TRD(B)=\sqrt{[0.97C+1.2]^2+[-0.89]^2}$$

For
$C=0 \Rightarrow TRD(B)=1.49$,
$C=0.5 \Rightarrow TRD(B)=1.9$,
$C=1 \Rightarrow TRD(B)=2.34$,
Then, $TRD(A)<TRD(B) \Rightarrow A<B$.

Example 6. Let $A=(0,0.4,0.6,0.8;1)$ and $B=(0.2,0.5,0.5,0.9;1)$ and $C=(0.1,0.6,0.7,0.8;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$I(A)=1.63C+0.94, \quad I(B)=1.1C+1.03, \quad I(C)=0.7C+1.2$$

* Step 2

$$D(A)=-1.3, \quad D(B)=-1.11, D(C)=-0.7$$

* Step 3

$$TRD(A)=\sqrt{[1.63C+0.94]^2+[-1.3]^2}$$

For
$C=0 \Rightarrow TRD(A)=1.6$,
$C=0.5 \Rightarrow TRD(A)=2.18$,
$C=1 \Rightarrow TRD(A)=2.88$,

$$TRD(B)=\sqrt{[1.1C+1.03]^2+[-1.11]^2}$$

For
$C=0 \Rightarrow TRD(B)=1.51$,
$C=0.5 \Rightarrow TRD(B)=1.93$,
$C=1 \Rightarrow TRD(B)=2.4$,

$$TRD(C)=\sqrt{[0.7C+1.2]^2+[-0.7]^2}$$

For
$C=0 \Rightarrow TRD(c)=1.39$,
$C=0.5 \Rightarrow TRD(c)=1.7$,
$C=1 \Rightarrow TRD(c)=2.02$,
Then, $TRD(A)>TRD(B)>TRD(C) \Rightarrow A>B>C$.

Example 7. Let $A=(0.1,0.2,0.4,0.5;1)$ and $B=(-2,0,0,2;1)$ be two generalized trapezoidal fuzzy number, then

* Step 1

$$I(A)=2.18C+0.65,\quad I(B)=-0.78C+0.78$$

* Step 2

$$D(A)=-1.03,\qquad D(B)=0.36$$

* Step 3

$$TRD(A)=\sqrt{[2.81C+0.65]^2+[-1.03]^2}$$

For

$C=0 \Rightarrow TRD(A)=1.22$,

$C=0.5 \Rightarrow TRD(A)=1.58$,

$C=1 \Rightarrow TRD(A)=3.61$,

$$TRD(B)=\sqrt{[-0.78C+0.78]^2+[0.36]^2}$$

For

$C=0 \Rightarrow TRD(B)=0.74$,

$C=0.5 \Rightarrow TRD(B)=0.53$,

$C=1 \Rightarrow TRD(B)=0.36$

Then, $TRD(A)>TRD(B) \Rightarrow A>B$.

For the validation of the proposed ranking function, in Table 1, it is shown that this approach is very simple and easy to apply in the real life problems.

Approaches	Example 1	Example 2	Example 3	Example 4	Example 5	Example 6	Example 7
Cheng [9]	A<B	A~B	Error	A~B	A>B	A<B<C	Error
Chu [10]	A<B	A<B	A<B	A<B	A>B	A<B<C	Error
Chen [7]	A<B	A<B	A<B	A<B	A>B	A<C<B	A>B
Abbasbandy [1]	Error	A~B	A<B	A~B	A<B	A<B<C	A>B
Chen [8]	A<B	A<B	A<B	A<B	A>B	A<B<C	A>B
Kumar [5]	A>B	A~B	A<B	A<B	A>B	A<B<C	A>B
Singh [24]	A~B	A>B	A>B	A~B	A~B	A>C>B	A<B
Rezvani [22]	A~B	A>B	A>B	A~B	A~B	A>C>B	A<B
Proposed approach	A<B	A<B	A>B	A>B	A<B	A>B>C	A>B

Table 1. A comparison of the ranking results for different approaches

5. Conclusion

Weighted average and weighted width of exponential fuzzy numbers combines in TRD distance for ranking method. A simpler and easier approach is proposed for the ranking of generalized trapezoidal fuzzy numbers.

REFERENCES

1. S.Abbasbandy and T.Hajjari, A new approach for ranking of trapezoidal fuzzy numbers. Computers and Mathematics with Applications, 57 (2009) 413-419.
2. T. Allahviranloo et al, a method foe ranking of fuzzy numbers using new weighted distance, Mathematical and Computational Applications, 16 (2011) (2) 359-369.
3. S. M. Baas and H. Kwakernaak, Rating and ranking of multiple-aspect alternatives using fuzzy sets, Automatica, 13 (1) (1977) 47-58.
4. G. Bortolan and R. Degani, A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems, 15(1) (1985) 119.
5. L. de Campos and G. A. Munoz, A subjective approach for ranking fuzzy numbers, Fuzzy Sets and Systems, 29(2) (1989) 145-153.
6. S.-H. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, Fuzzy Sets and Systems, 17(2) (1985) 113-129.
7. S.J.Chen and S.M.Chen, Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers, Applied Intelligence, 26 (2007) 1-11.
8. S.M.Chen and J.H.Chen, Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Expert Systems with Applications, 36 (2009) 6833-6842.
9. C.H.Cheng, A new approach for ranking fuzzy numbers by distance method. Fuzzy Sets and Systems, 95(3) (1998) 307-317.
10. T.C.Chu and C.T.Tsao, Ranking fuzzy numbers with an area between the centroid point and original point. Computers and Mathematics with Applications, 43(1-2) (2002) 111-117.
11. M. Delgado, J. L. Verdegay, and M. A. Vila, A procedure for ranking fuzzy numbers using fuzzy relations, Fuzzy Sets and Systems, 26(1) (1988) 49-62.
12. D. Dubois and H. Prade, The mean value of a fuzzy number, Fuzzy Sets and Systems, 24(3) (1987) 279-300.
13. R. Jain, Decision making in the presence of fuzzy variables, IEEE Transactions on Systems, Man and Cybernetics, 6(10) (1976) 698-703.
14. K. Kim and K. S. Park, Ranking fuzzy numbers with index of optimism, Fuzzy Sets and Systems, 35(2) (1990) 143-150.
15. A.Kumar, P.Singh, A.Kaur and P.Kaur, RM approach for ranking of generalized trapezoidal fuzzy numbers, Fuzzy Information and Engineering, 2 (1) (2010) 37-47.
16. E.S.Lee and R.-J. Li, Comparison of fuzzy numbers based on the probability measure of fuzzy events, Computers and Mathematics with Applications, 15(10) (1988) 887-896.
17. C.Liang, J.Wu, and J.Zhang, Ranking indices and rules for fuzzy numbers based on gravity center point. Paper presented at the 6th world Congress on Intelligent Control and Automation, Dalian, China, (2006) 21- 23.
18. S. Rezvani, Graded mean representation method with triangular fuzzy number, World Applied Sciences Journal, 11(7) (2010) 871-876.
19. S. Rezvani, Multiplication operation on trapezoidal fuzzy numbers. Journal of Physical Sciences, 15 (2011) 17-26.

20. S. Rezvani, A new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers, International Journal of Applied Operational Research, 2(3) (2012) 83-90.
21. S. Rezvani, A new approach ranking of exponential trapezoidal fuzzy numbers, Journal of Physical Sciences, 16 (2012) 45-57.
22. S. Rezvani, A new method for ranking in areas of two generalized trapezoidal fuzzy numbers, International Journal of Fuzzy Logic Systems, 3(1) (2013) 17-24.
23. S. Rezvani, Ranking method of trapezoidal intuitionistic fuzzy numbers, Annals of Fuzzy Mathematics and Informatics, 5(3) (2013) 515-523.
24. P. Singh et al, ranking of generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread, Turkish Journal of Fuzzy Systems, 1(2) (2010) 141-152.
25. Y.J.Wang, H.S.Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points. Computers and Mathematics with Applications, 55 (2008) 2033-2042.
26. J.Yao and K.Wu, Ranking fuzzy numbers based on decomposition principle and signed distance, Fuzzy Sets and Systems, 116 (2000) 275-288.
27. L. A. Zadeh, Fuzzy sets, Information and Control, 8(3) (1965) 338-353.